Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 223 FEBRUARY, 1937 No. 2

PHOTO-ELECTRIC EMISSION.*

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NOTE.

The advantages of using the method previously described for preparing clean surfaces for the study of photo-electric emission from different metals are briefly discussed. Experiments are described which show that when certain metal surfaces react with oxygen the shift in sensitivity toward the longer wave-length is due to a reaction and not to the physical nature of the surface and that the similar shift when an alkali metal deposits on another metal is likewise the result of a reaction.

INTRODUCTION.

The commercial use of photo-electric cells has been greatly extended in recent years. To meet these demands two distinct advances in the art of producing active surfaces were necessary, namely the development of surfaces of the different metals from which constant and reproducible emission is obtained when excited by a fixed intensity and quality of light and second the production of surfaces which respond to longer wave-length radiations than were previously obtainable. The former was possible by the use of a technique for preparing active surfaces free from contamination so that they remain clean during use. The second result is brought about by the development of specific methods for treating

* Based on a paper presented by Dr. Rentschler at the Stated Meeting held Wednesday November 20, 1935.

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the photo-electrically active surface whereby it becomes sensitive to radiations of longer wave-lengths than those to which it responded before activation. This latter development resulted in the advancing of theories to explain the observed results which are important in explaining both photo-electric and thermionic emission from activated surfaces. It is the object of this paper to outline certain experiments and the results obtained which have an important bearing on the theories previously advanced to account for the increased emission from these activated surfaces.

PREPARATION OF CLEAN METAL SURFACES.

In 1932[1] a method was first described by which it is possible to produce remarkably uniform, reproducible and constant surfaces of not only the more stable metals such as nickel, iron, etc., but of such highly active metals as uranium, thorium, etc., as well. The metal in wire or rod form is mounted as shown by *w* in Fig. 1. A frame *b* is mounted on a collar clamped around the stem *c* and has the electrode *d* mounted on it as shown. The wire *e* is sealed into the upper end of the bulb and connects with the cathode *d*. A cylindrical sleeve *f* slides on the frame and protects the glass or quartz container during the sputtering process used in preparing the active surface. The bulb is sealed to a pump at *k*, exhausted and baked in the usual manner and the metal parts outgassed by high frequency heating. The bulb is then filled with pure argon at about 2 mm. pressure and sealed off. A discharge is then passed through the argon with the wire *w* the negative terminal. Any active residual gas impurity in the argon is cleaned up by this discharge and finally additional sputtered metal from the wire forms a layer on the plate *d* which serves as the cathode of the photocell. The sputtering process requires from 20 to 50 hours depending upon the discharge current which is never great enough to heat the bulb appreciably above room temperature so that the further liberation of water vapor, etc., from the glass is avoided. The final exhaustion of the argon is preferably accomplished by having a thoroughly exhausted and baked charcoal bulb sealed to the cell at *g* in the original construction of the cell. After the sputtering is completed,

FIG. 1.

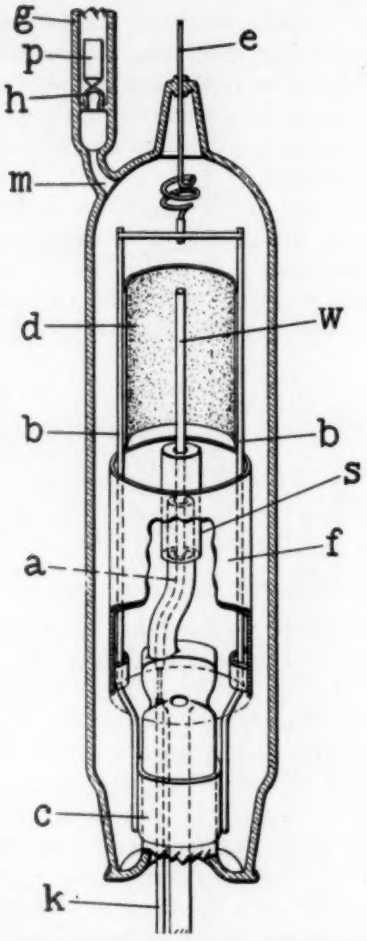

Photo-cell construction for preparing active surface of any metal.

the charcoal is immersed in liquid air, the thin glass partition *h* is broken by the plunger *p* and the argon taken up by the charcoal after which the cell is sealed off at *m*.

This charcoal exhaust is preferable to again sealing the cell to a pump as previously described since the connecting tube from the cell to the pump cannot be so thoroughly torched that some deleterious gas or vapor may not again

diffuse into the cell during the argon exhaust. Cells made in this manner have been in constant use for several years without showing noticeable decrease in sensitivity. Different cells made from the same metal are uniform in sensitivity and have the same threshold wave-length showing that a highly satisfactory duplication of photo-electric characteristics is possible by this method.

The photo-electric work functions calculated from the measured threshold wave-lengths for tungsten, molybdenum, tantalum and platinum cells prepared in this manner agree with the thermionic work functions of these same metals reported by other observers as the following table shows.

TABLE I.

	Photo-electric Work Function	Thermionic Work Function
Tungsten	4.60 volts[1]	4.54 volts[2]
Tantalum	4.12 volts[1]	4.12 volts[3]
Molybdenum	4.33 volts[3]	4.31 volts[2]
Platinum	Higher than 6.17 volts[5]	6.27 to 6.30 volts[4]

ACTIVATION OF CLEAN SURFACES.

With a satisfactory method for producing constant and reproducible active metal surfaces the study of the activation of different surfaces to produce greater sensitivity and a shift in the photo-electric response toward the longer wave-length is greatly simplified. It has been known for some time that when an oxidized silver or copper surface and an alkali metal are allowed to react in a definite manner the photo-electric threshold of the surface is shifted toward the longer wave-length. This shift is commonly explained as being due to the formation of a monomolecular layer of the alkali metal on an oxide on the silver or copper.[6, 7] To obtain some information as to the manner in which the presence of the oxygen causes the threshold shift small amounts of oxygen were introduced into cells with different metals in the following manner.[5] The active surface of the metal in question was prepared as described under preparation of clean metal surfaces. A tube *T* as shown in Fig. 2 was attached to the cell. A small glass container *C* of known volume and filled with pure dry oxygen at a known pressure was inserted in the side tube *t*.

FIG. 2.

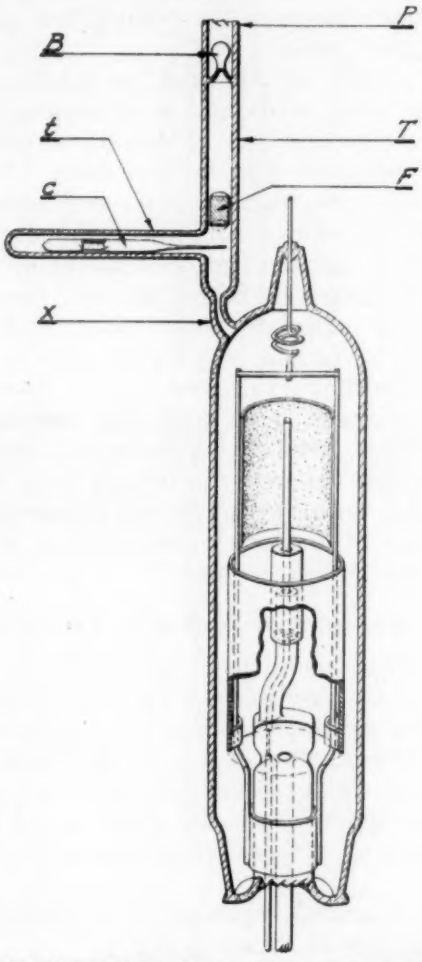

Photo-cell with side tube for introducing a definite small amount of oxygen.

The completed cell (the argon exhaust tube is not shown in Fig. 2) was tested for spectral sensitivity, after which the thin tip of the oxygen tube was broken by means of the plunger *F* and the oxygen permitted to react with the active surface. The sensitivity was again tested and finally the cell was again sealed to a high vacuum pump at *P*, the connection to the

pump carefully exhausted, and the partition B broken by means of an iron plunger in the exhaust line above B, sealed off at X and again tested.

For such metals as titanium, molybdenum, tungsten, iron, etc., any small introduction of oxygen decreased the sensitivity and lowered the threshold wave-length. This is the normal photo-electric fatigue effect. Gold does not react with oxygen and there was no noticeable effect when oxygen was added. For the highly chemically active metals thorium, uranium, calcium and caesium the introduction of a small amount of oxygen shifts the threshold toward the longer wave-length while materially larger amounts of oxygen again produce the normal fatigue effect, that is lower the threshold wave-length below that of the clean metal. Thorium, uranium and calcium, however, have such high melting points and such low volatilities that the shifts appear certain to be the result of a reaction between the oxygen with the metal and not due to the formation of a monomolecular layer. A similar lowering of the work function for magnesium by oxygen has recently been reported.[8]

PHOTO-ELECTRIC EMISSION FROM ALKALI METAL FILM ON ANOTHER METAL.

When an alkali metal such as sodium or potassium is allowed to deposit slowly on a supposedly clean metal to form a thin film the threshold wave-length is longer than for a thick layer of the alkali metal.[9] To determine whether this shift in sensitivity is due to the thin layer of the alkali metal or to some reaction between the alkali metal and some impurity remaining on the base metal the following tests were made.

Two bulbs K and T separated by a thin glass partition were sealed together as shown in Fig. 3.

Charcoal tubes C_1 and C with thin partitions B_1 and B were sealed to the bulbs at d_1 and d. The charcoal in these tubes was baked and the tubes exhausted by first sealing them to pumps at a_1 and a. The bulb K with a cup M containing a mixture of potassium dichromate and silicon was sealed to a pump at L_1 exhausted and baked. The cup M was then heated by high frequency to generate the potassium and finally sealed off at L_1. The phototube ele-

FIG. 3.

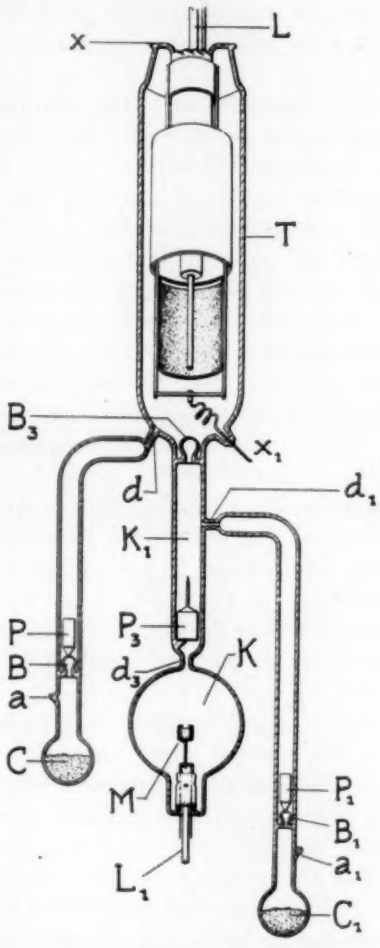

Photo-cell for investigating the change in sensitivity as an alkali metal slowly condenses on a metal.

ments were then sealed into the bulb T at X and X_1. The cell T was sealed to the pump at L, thoroughly exhausted and baked and the elements degassed by high frequency heating. After all parts were cool the potassium in K was distilled into the compartment K_1 and the bulb K sealed off at d_3. The charcoal bulb C_1 was then placed in liquid air,

the partition B_1 was broken by operating the plunger P_1 and the potassium distilled into the upper part of K_1 mostly into the bulb B_3. After the glass was cool the bulb C_1 was sealed off at d_1.

The cell T was then filled with argon and sealed off at L. The active surface of the metal under test was then prepared by sputtering as previously described. The charcoal bulb C was then immersed in liquid air and the partition B broken and after all the argon was absorbed by the charcoal the bulb C was sealed off at d. This detailed procedure was followed to eliminate contaminations as far as possible.

The sputtered surface was first tested for sensitivity to different wave-lengths as a check. The plunger P_3 was then used to break the partition B_3 and the potassium allowed to vaporize at room temperature and slowly deposit on the clean metal cathode. The light from a tungsten incandescent lamp was diffusely focussed on the cathode surface and the sensitivity measured in the usual manner. At room temperature the distillation of the potassium proceeds slowly and a test generally extended over a period of several hundred hours. The sensitivity of the surface for the light through different filters was noted at regular intervals over this period. The relative position of the light and the cell was fixed throughout the test.

Surfaces of silver, gold, copper and molybdenum were tested with the same general result, namely, that the sensitivity to light passing through Wratten filters No. 88 and No. 70 which do not transmit radiations shorter than 6900 A.U. and 6500 A.U. respectively was almost negligible for all surfaces prepared in this manner. A molybdenum plate was next mounted in a bulb, exhausted, and heated by electron bombardment until the entire bulb was covered with a deposit of evaporated molybdenum. The plate thus cleaned was allowed to cool and again mounted in a new bulb and again bombarded as long and as hot as possible without blackening the bulb. The molybdenum so treated was far less sensitive than a sputtered molybdenum surface and when potassium was slowly deposited on this bombarded molybdenum the maximum sensitivity for the light through the same filters No. 88 and No. 70 was between 50 and 100 times as great as when

FIG. 4.

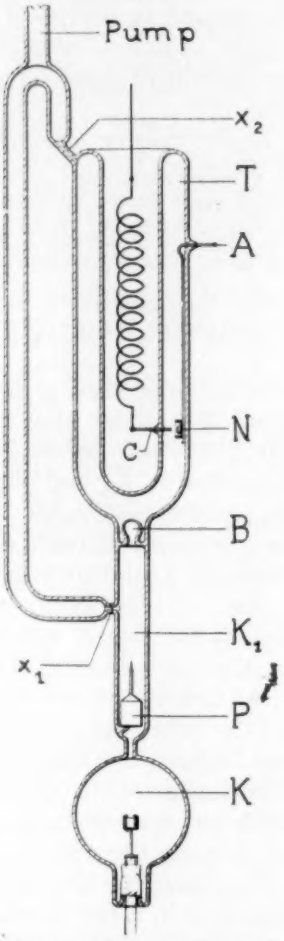

Photo-cell construction used for determining the change in sensitivity as an alkali metal condenses on another metal maintained at liquid air temperature.

deposited on the sputtered surface of molybdenum. Surfaces of copper and molybdenum which were cleaned only by high frequency heating on the pump show no increased sensitivity as the potassium deposited on the metal. A surface of silver cleaned by high frequency heating only similarly shows a decidedly greater response as potassium deposits on it than a

sputtered silver surface. The results of all these tests indicate that if a metal surface could be prepared absolutely free from all contaminations such as is unavoidable during tipping off, etc., there would be no shift of sensitivity as the alkali metal deposits on the metal.

As a final test of the theory advanced to explain these results surfaces of magnesium and zinc were prepared by distilling these metals and then allowing the potassium to slowly condense on them in a manner similar to that used above. The results were like that obtained when potassium was allowed to condense on molybdenum cleaned by bombardment. These tests were then repeated in the following manner.

A double-walled container similar to a liquid air bottle was used to construct the cell as shown in Fig. 4. A wire C through the inner glass bulb serves as the cathode lead. The potassium compartment K_1 and the phototube T were sealed to a pump, exhausted and baked in the usual manner. The potassium was generated in bulb K by high frequency heating the cup containing a mixture of potassium dichromate and silicon. The zinc or magnesium was then vaporized by induction heating of the cup N welded to the anode lead A and deposited on the inner bulb making contact with the cathode lead C. The potassium was then distilled into the compartment B and the tubes sealed off at X_2 and X_1. The partition B was then broken, the inner bulb filled with liquid air and the potassium allowed to distil on to the cathode maintained at liquid air temperature, and the change in sensitivity to light through the different filters measured at regular intervals. To reduce the time required for a complete test from several hundred to a few hours the potassium container was placed in a heater and maintained slightly above room temperature. As long as liquid air was maintained in the inner bulb there was no detectable response to light passing through either of the two filters No. 88 or No. 70 while the total response without filters approached a constant value. This indicates that the shift in sensitivity when an alkali metal deposits on another metal is due to a reaction between the alkali metal and some impurity on the base metal and not due to the physical nature of the deposit that is a thin film effect.

CONCLUSIONS.

The shift in the photo-electric threshold toward the longer wave-length when certain metal surfaces such as thorium, uranium, the alkali and alkaline earth metals are sensitized by an oxygen treatment is due to a reaction, probably the formation of a sub-oxide. A similar shift in sensitivity when an alkali metal is deposited in a thin film on another metal is due to a reaction between the alkali metal and some impurity on the base metal and not due to the physical nature of the deposit.

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Coffee Chaff Violates Federal Food and Drug Act.—The adulteration of coffee with coffee chaff, a waste product of the coffee-roasting industry, has from time to time brought the products of coffee dealers into conflict with the Federal Food and Drugs Act. A recent action instigated by the U. S. Food and Drug Administration has resulted in a plea of guilty, with a fine of $50 and costs. A manufacturer had been furnishing this adulterant to coffee dealers under the name "coffee screenings." The chaff was purchased from the trade under fictitious firm names. After further roasting and molding in a form resembling ground roasted coffee it was sold back to the trade under the manufacturer's real name. The adulteration provisions of the law prohibits the sale for food purposes of this valueless waste product, coffee chaff.

R.

Farmers Care for Timber as Conservation Measure.—(U. S. Dept. of Agric. Clip Sheet.) Farmers in many parts of the country are beginning to pay more attention to the care of their farm woodlands as a measure of soil and moisture conservation and more profitable timber production, says the Soil Conservation Service. It cites, for example, the case of a woman who operates a farm near Lexington, N. C., in a demonstration area of the Service. Her woodland comprises 10 acres, and one section of the plot consists largely of mixed oaks, mainly white, red, and post oaks. Another section is mainly in shortleaf pine and red cedar. Sleet storms had damaged the pines and cedars greatly, but the oaks fared somewhat better.

As a part of the demonstration, Conservation men got to work and removed the pines when the tops were so badly broken by the storms that growth had practically stopped and decay had set in. Cedars, which were badly bent by the wind and showed no signs of straightening, were cut out for fence posts. Work did not stop there. Dogwood and various shrub species which supply food for game birds and other wildlife were left standing. Some of the gums, sourwoods, and red maples were cut out, especially when they were interfering with the growth of the more desirable trees.

That woodland is now in much better condition. The danger from fire and from insect and plant-disease attacks has been greatly reduced. Remaining trees now may grow more rapidly since the competition is not so keen, and the spaces left by trees cut out supply a fertile seed bed for a future crop.

R.

MENDEL AND HIS PLACE IN THE DEVELOPMENT OF GENETICS.*

BY

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Several years ago the *Journal of Heredity* ran a frontispiece with the caption "Here is where Genetics began." The picture showed a small garden inside the Augustinian Monastery of St. Thomas in the town of Brno, Czechoslovakia. It was in this garden that Gregor Johann Mendel performed his experiments of crossbreeding the edible pea which gave us our first exact knowledge of the modes of inheritance of certain characteristics.

The town of Brno has undergone many changes since Mendel worked there in the sixth decade of the nineteenth century. It is an old town with many historic associations. In the early days it was an important city in the Kingdom of Moravia. Known by its German name of Brünn, Napoleon slept here on the night after his famous victory on the nearby fields of Austerlitz. For many years an important city of the Austro-Hungarian Empire, it now finds itself as the second city of the new Czechoslovakian Republic. And with the wave of nationalism of today, the Czech form of the name—Brno—is insisted upon.

The town lies about the foot of an isolated hill called the Spielberg which is crowned by an old fortress to which, until the early days of the reign of the Emperor Franz Joseph, were consigned the political prisoners of the Empire. It is said that the great German General von Moltke smiled but twice in his life—once when he heard of the death of his mother-in-law and the second time when he saw the defenses of the Spielberg at Brno.

At the foot of this hill lies the Augustinian Monastery of St. Thomas with its beautiful church and its interesting and

* Presented at the Stated Meeting held Wednesday, October 21, 1936.

stately cloister. The buildings date back to 1322 when they first housed a nunnery founded by Queen Elisabeth, the widow of Wenceslaus and of Rudolf, Kings of Bohemia. Because of its patron, it was called the Königinkloster. Little or nothing remains of the original buildings—most of the present cloister dates only to 1762 when an extensive restoration was effected. The order of St. Thomas is also ancient, dating back to 1359 but transferred to the Königinkloster in Brno only as late as 1793. It was in an enclosed garden that Mendel performed his painstaking experiments.

Last year the American Genetic Association published a list of definitions. In these, Genetics is described as "the science of heredity, variation, sex determination and related phenomena." As usual this definition, like most definitions, is not particularly informative. The importance of the science of genetics cannot be overemphasized. The intelligent individual, without special interest and special training, will think at once of the scientific breeding of animals or of Luther Burbank's scientific development of fruits and flowers. One thinks of the inheritance of physical characteristics in man—of stature, eye color, hair color and the like. But careful experiments in the last fifty years indicate that many other characteristics are heritable as well as these more obvious physical ones. Breeders of dogs will now tell you that it is possible by selection to breed for temperament as well as for physical form. We know that certain kinds of feeble-mindedness and insanity are inherited in certain ways. We know that either susceptibility or relative immunity to certain diseases may be inherited. Within the last six months there has been formed the British National Human Heredity Committee,[1] including such men as Grafton Elliott-Smith and Arthur Kieth, whose statement begins as follows: "Problems of national health have reached a point where the hereditary element can no longer be neglected. The leaders of the medical profession are no longer satisfied with the alleviation of disease, but are acutely conscious of the need for fuller knowledge of heredity in connection with its prevention. This applies not only to preventing the transmission of defects. It is recognized that methods of cure

[1] Reported in *Science*, 1936, 83, 543 f.

must vary with the type of constitution of the patient, and in this connection information concerning heredity is of great importance. In education, in training, and in choice of career, the ascertainment of innate endowment not only prevents waste and failure but would contribute largely to the attainment of success. . . . It is strange to think that students of fruit-flies, or mice, have at their command the latest information, while those similarly concerned with man can look nowhere for a complete survey of the knowledge they require."

In these days when there is so much emphasis in certain countries regarding race and racial purity, it is obvious that problems of genetics must loom important. Here is a complex problem indeed as indicated by the anthropologist Earnest A. Hooton,[2] who defines a race as a "physical division of mankind, the members of which are distinguished by the possession of similar combinations of anatomical features due to their common heredity." He further complicates the problem by asserting "a pure race is little more than an anthropological abstraction; no pure race can be found in any civilized country. Racial purity is restricted, at best, to remnants of savage groups in isolated wildernesses. The present races of man have intermingled and interbred for many thousands of years, so that their genealogical lines have become inextricably confused." And again: "The composite origin of most of the existing races of man is demonstrable. Thus the Polynesian represents a stabilized blend of white, Negroid and Mongoloid elements. The so-called Nordic race is probably a hybrid derivative of several strains present in Europe during the glacial period, to which have been added in historic times, Alpine, Mongoloid and other racial elements (carried by Lapps, Finns, Slavs and other peoples who have mixed with the inhabitants of the "Nordic" area)."

Problems of human heredity are thus seen to be exceedingly complex and to be essentially in the class of other problems of hybridism. One can buy a rat from the Wistar Institute at Anatomy with a known heredity of 100 generations and more. Contrasted to this, it is said that there are not three families in England today who can trace a single

[2] E. A. Hooton, "Plain Statements about Race," *Science*, 1936, **83**, 511–513.

unbroken line of their genealogy to the Conquest and that there is not a single family which can trace its lineage back to Saxon times. How many of my listeners can name *all* of their great-grandparents—back only three generations? And I doubt if any one of you has a record of the physical, mental and temperamental characteristics of these same eight great-grandparents.

Thus problems of human heredity bring us a long way from the breeding of red color in dachshunds or the artificial production of the loganberry. Indeed, one might say that the science of genetics is as important and as propaedeutic to all of the biological sciences as statistics is for all quantitative science in general. Besides the zoologist and the botanist, who have been in the forefront of the development of genetic science, it is important for the anatomist, the physiologist and all of the rest of the sciences on which medicine is based, as well as for the anthropologist and the psychologist.

And the *Journal of Heredity* showed a picture of Mendel's garden with the statement that "Here is where Genetics began." Naturally Mendel's work was not without historic background. Before his time there had been a number of experiments in hybridization. At one time it was believed that the ostrich was the result of a crossed mating between a camel and a sparrow! One authority [3] believes that hybridization and sex differences in plants was known by Shakespeare and he points to a passage in Act IV, Scene 3 of "A Winter's Tale" believing that this passage cannot be otherwise interpreted.

Professor Zirkle [4] has gathered together many early records of botanical hybridization. Camerarius in 1694 is usually credited with being the first to prove experimentally that pollen is necessary for seed development—thus establishing the fact of different sexes in plants. Koelreuter in 1761 is usually credited with the utilization of Camerarius' observation in making the first systematic study of plant

[3] W. E. Praeger, "Did Shakespeare Know Plant Hybrids?" *J. Heredity*, 1932, **23**, 161 f.

[4] C. Zirkle, "Some Forgotten Records of Hybridization and Sex in Plants," 1716–1739, *J. Heredity*, 1932, **23**, 433–448; "More Records of Plant Hybridization Before Koelreuter," *J. Heredity*, 1934, **25**, 3–18.

hybridization and, in so doing, made the very important observation that both parents contribute to the form of the offspring. Apparently, Fairchild, a gardener in London, artificially crossed two species of pink as early as 1719, or over forty years before the Koelreuter observations. Actually Professor Zirkle has found a letter written by Cotton Mather dated 1716, or three years before the Fairchild observation, in which he describes the hybridization of different species of Indian corn and of squashes with gourds. These, of course, are chance observations of cross fertilization and are not carefully planned experiments.

One could add to the list of names of those who made early observations of this sort. There are the bi-sexual observations of plants by Philip Miller; the botanical observations of such men as Paul Dudley, James Logan (a Colonial Governor of Pennsylvania), our own John Bartram of Philadelphia and many others.

After the work of Koelreuter, both amateur and professional botanists produced many hybrids but no one systematically seems to have planned experiments which would indicate the *mechanism* for the resemblance of the offspring to the parental forms. True in 1861 the French Academy offered a prize "For the study of plant hybrids from the point of view of their fertility and of the perpetuation of their characters." Writing for this prize, Naudin clearly enunciated what we know as the concept of the dissociation of characters of cross-breds in a paper which was published in 1865.

Mendel's own experiments were published the same year as the appearance of the work of Naudin, namely in 1865. Mendel had shown an early interest in problems of evolution and of hybridization. He obtained copies of the works of Charles Darwin immediately on publication; he owned a copy of Erasmus Darwin's "Zoonomia." The copy of Darwin's "On the Various Contrivances by Which Orchids Are Fertilized by Insects," published in 1862, is now in the library of the Augustinian Monastery at Brünn and contains many marginal notations in Mendel's handwriting.

In 1856, at the age of thirty-four years, Mendel began his experiments in crossing the edible pea—the first experiments

which he carried on intensively for seven years until 1863. These experiments were most carefully planned. He recognized that any hybridization experiment, to be successful, must utilize plant or animal forms which have the following specifications: (1) They must have different forms distinguishable by sharply defined and constant characters and (2) during the season of the flowering, it must be possible to protect the flowers easily from chance fertilization with undesired pollen. The edible pea beautifully fulfils both of these requirements.

From various sources, Mendel collected 34 different kinds of pea seeds. For two years he tested the strains for purity and finally selected 22 varieties as suitable for his experiments. He then ignored those characteristics which differed merely in degree and confined his attention to certain paired characteristics where a really sharp distinction could be made.

This development of method is perhaps Mendel's greatest contribution to genetic theory and practice because it alone made possible the statistical approach to this problem which was the only possible means for the formulation of his conclusions. By pairing his characteristics and, in each hybridization, working only with a single pair, he avoided the "shot gun" methods of former investigators who had worked with a number of characteristics simultaneously, so that frequently the results of one characteristic were masked by the presence of others.

Mendel finally selected seven pairs of characteristics as the problem of his experiments. They were:

1. The shape of the ripe seed—either round and full or irregularly angular and deeply wrinkled.
2. The difference in the color of the seed cotyledons—yellow or green.
3. The difference in the color of the seed—the coat either white or grey to buff.
4. The difference in the shape of the ripe pods—either constricted between the seeds or simply inflated.
5. The difference in the color of the ripe pods—either dark green or yellow.
6. The difference of the position of the flowers on the stem—either axial or terminal.

7. The difference in the length of the stem—either the "dwarf" or the nearly five times as long "tall" plant.

The specimen here this evening is unquestionably one illustrating either one of the last two pairs of characteristics. If it was saved by Mendel to illustrate the sixth pair of characteristics studied, it is an example of the terminal position of the flowers. If it was saved to illustrate the seventh pair of characteristics, it is an example of the "dwarf" variety. I rather suspect that it illustrates this characteristic because there are preserved in the Monastery at Brno several specimens which most certainly illustrate the tall variety of plant.

Careful record was kept of all of the various stages of the experiment. In the first stage of cross breeding of the seven different characteristics, Mendel made 287 fertilizations on 70 plants. Then came the stages of breeding the offspring of the first generations again by cross fertilization and again with careful records of all of the offspring. Some idea of the magnitude of these experiments may be conceived from the published statement of Mendel which says: "Among more than 10,000 plants which were carefully examined, there were only a very few cases where an indubitable false impregnation had occurred."

Mendel also worked with an eighth characteristic, namely, the flowering season, but these observations were not completed when he published his results.

Taking two vigorous plants which differed markedly in respect to one of the several characteristics, Mendel crossed them mutually, that is, he pollinated the first with the second and also the second with the first. He found that the result was the same in either case. The offspring of this first crossing is the first generation to which the symbol F_1 has been given. It was found that all offspring from either parent resembled one particular parent with regard to the characteristic studied and that no offspring from either parent resembled the other. This differentiation gives the basis between considering a characteristic either "dominant" or "recessive." All offspring of the first generation exhibited the dominant characteristic of the pair and none were in the form of the recessive. And the dominant form for each of

the seven pairs of characteristics chosen by Mendel for study, no matter from which of the pair of plants they were grown, proved to be:

1. The round form of seed
2. Yellow color of the cotyledons
3. Grey color of the seed coat
4. Rounded shape of pod
5. Green coloring of ripe pod
6. Axial distribution of flowers on the stem
7. Greater length of stem—the "tall" variety of plant.

Indeed, with regard to the last characteristic, Mendel says: "It must be stated that the longer of the two parental stems is usually exceeded by the hybrid. . . . Thus, for instance, in repeated experiments, stems of 1 foot and 6 feet in length yielded without exception hybrids which varied in length between 6 feet and 7½ feet."

In the first generation bred from the hybrids, to which the symbol F_2 has been applied, the results were quite different. These results were obtained from 19,752 observations of pod, seed, stem and flower from 5,123 plants grown for this second stage of the experiment.

Mendel found that while, in the generation F_1, all offspring were of the dominant form, in the second generation F_2, some were clearly of the dominant form, some of the pure recessive form and others were intermediate but approaching the form of the dominant grandparent. Mendel kept complete records and had sufficient data so that he was able to work out the ratios between the dominant and recessive characteristics of the F_2 generation. Hence, for the first pair of characteristics, he was able to say: "From 253 hybrids, 7,324 seeds were obtained in the second trial year. Among them were 5,474 round or roundish ones and 1,850 angular wrinkled ones."

Thus, for this characteristic, the ratio of dominant to recessive was 2.96 to 1. The smallest ratio which he obtained in this second generation for any of the 7 pairs of characteristics was 2.82 to 1 and the largest ratio was 3.15 to 1. The average of all of the seven ratios turned out to be slightly greater than 2.98 to 1,—a result which is a surprisingly close value to the ideal 3 to 1 ratio which one might expect considering the size of the experimental data.

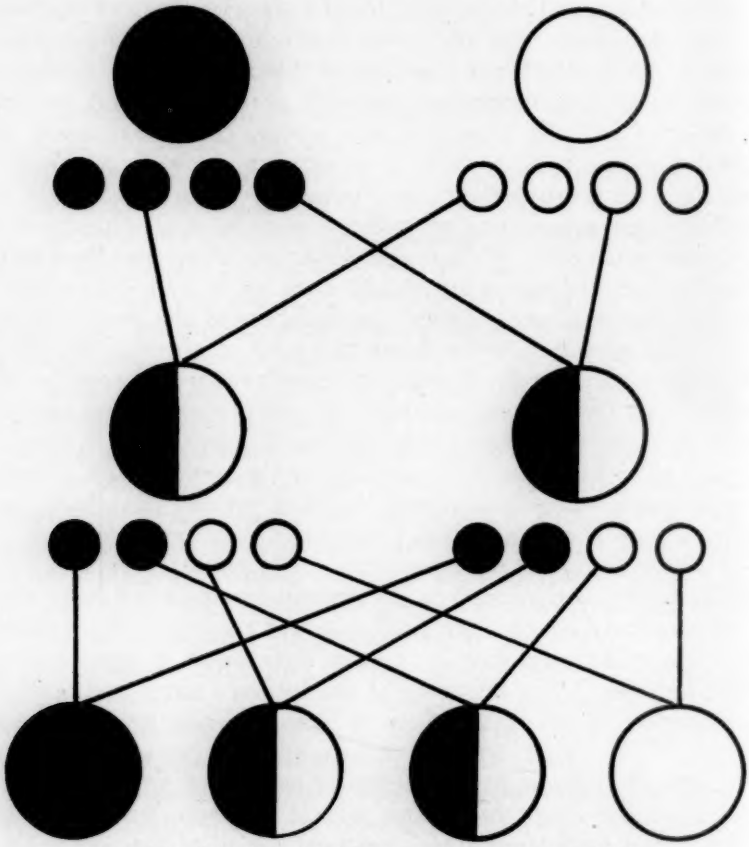
The next year Mendel raised the seeds from this second hybrid generation and records the results from 1250 plants. The results of F_3 were found again to be different from those of either F_1 or F_2. The statistical analysis of the results indicates that one-fourth of the plants produced characteristics of the true original dominant type; one-fourth produced characteristics of the true original recessive and one-half of the plants showed characteristics of various degrees of mixture of the dominant and recessive characteristics. From these results, he worked out his famous formula of the ultimate relations of dominant and recessive being expressed by the ratio 2 : 1 : 1. In Mendel's own words, this is expressed as follows: "The ratio of 3 to 1, in accordance with which the distribution of dominant and recessive characters results in the first generation resolves itself, therefore, in all experiments into the ratio of 2 : 1 : 1 if the dominant character be differentiated according to its significance as a hybrid-character or as a parental one. Since the members of the first generation (F_2) spring directly from the seed of the hybrids (F_1) it is now clear that the hybrids in respect to two differentiating characters form seeds one-half of which develop again the hybrid form, while the other half yield plants which remain constant and receive the dominant or the recessive characters respectively in equal numbers." Here, in two sentences, are summed up the principal findings of these experiments.

But Mendel did not stop here. The report indicates that experiments were carried on for characteristics 1 and 2 through 6 generations; for characteristics 3 and 7 through 5 generations and for characteristics 4, 5 and 6 through 4 generations. And he says: "The offspring of the hybrids separated in each generation in the ratio 2 : 1 : 1 into hybrids and constant forms." In other words, the experiment was carried far enough and through a sufficient number of generations to demonstrate that this ratio, which we now know as the Mendelian Ratio, remained constant for all of the succeeding generations after F_2.

One may properly think of these relationships in terms of pure chance. If one tosses two coins simultaneously, the laws of probability tell us that we may expect both to turn heads in one-fourth of the trials; both to turn tails in one-

fourth of the trials, and for a head and tail to occur in one-half of the cases. This is exactly the Mendelian 2 : 1 : 1 ratio. Consider the heads to be dominant and the tails to be the recessive characteristics and one can, at once translate the pure chance of the coin tossing experience into the genetic expectation.

FIG. 1.



The formal relationships of the inheritance of two paired characteristics.

These relationships are illustrated schematically and purely formally for the inheritance of a single characteristic in Fig. 1. In the top row, the characteristic in the left hand circle is represented in black and the paired characteristic in

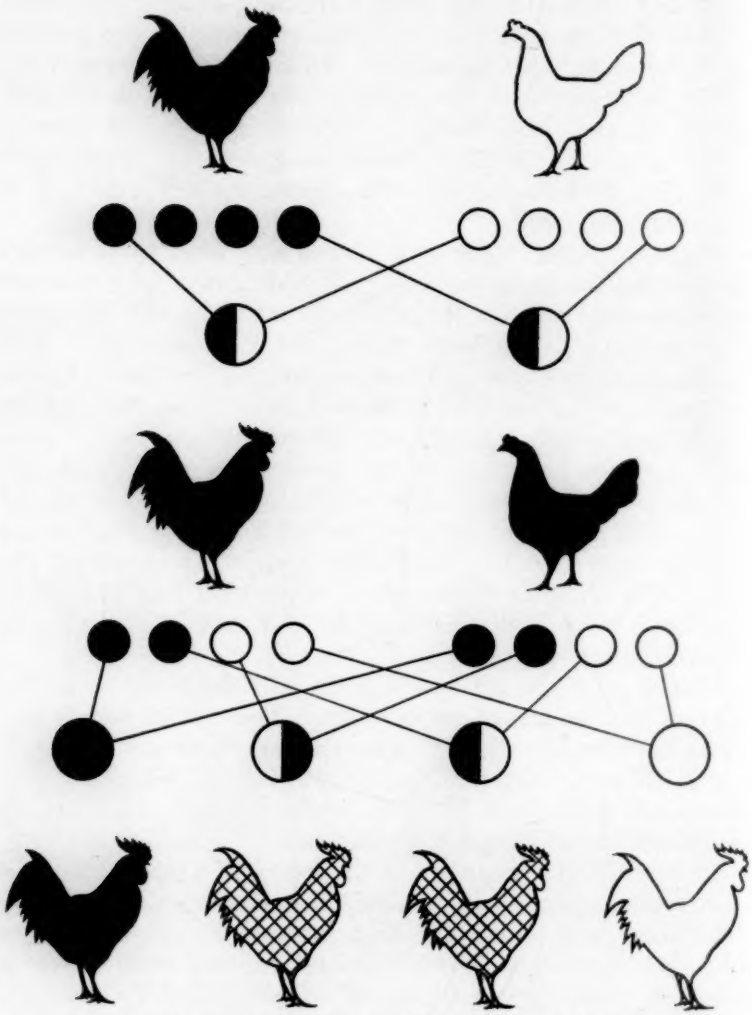
the other parent by a white circle. Directly below each parent are four small circles representing the gametes—the reproductive cells—from each parent. The number of gametes in this case is unimportant. The essential fact is that all gametes of the black parent are black and that all gametes of the white parent are white. Hence if any gamete from either parent combines with any gamete from the other parent, then the genetic constitution of all of the offspring of the generation F_1 will and must be mixed. This fact is represented by the two large circles in the center of the chart which are both half white and half black to indicate this mixed genetic constitution. The actual form of this generation with respect to this pair of characteristics will depend upon whether black or white is dominant.

Directly below these two circles are four small circles representing the gametes of F_1. In both cases, four gametes are represented and, in this case, the number will be found to be important. In both cases, it will be observed, that two gametes are black and two are white. By pure chance, we may expect these two groups of four circles to combine as indicated by the lines drawn downward from them—thus producing four offspring of the generation F_2. These are represented by the four large circles at the bottom of the chart. And it can be seen that the left hand circle will be all black (because it is produced only from black gametes); the two middle circles will be mixed white and black and the right hand circle will be all white (because it is produced only from white gametes).

This is pure chance and it is to be thought of as in the realm of mathematical probability. This chart merely describes the genetic expectation for the inheritance of the single pair of different characteristics. In Fig. 2, which exactly corresponds to the first in form, is what actually happens for the genetic development of what has been called the Blue Andalusian Fowl. Starting at the top with a pure black and a pure white strain, in which all of the gametes are respectively solely black and white, the first mixture F_1 gives offspring which are essentially black because, in this case, black is dominant. The crossbreeding of these hybrids to give the generation F_2 gives the results indicated at the bottom of the

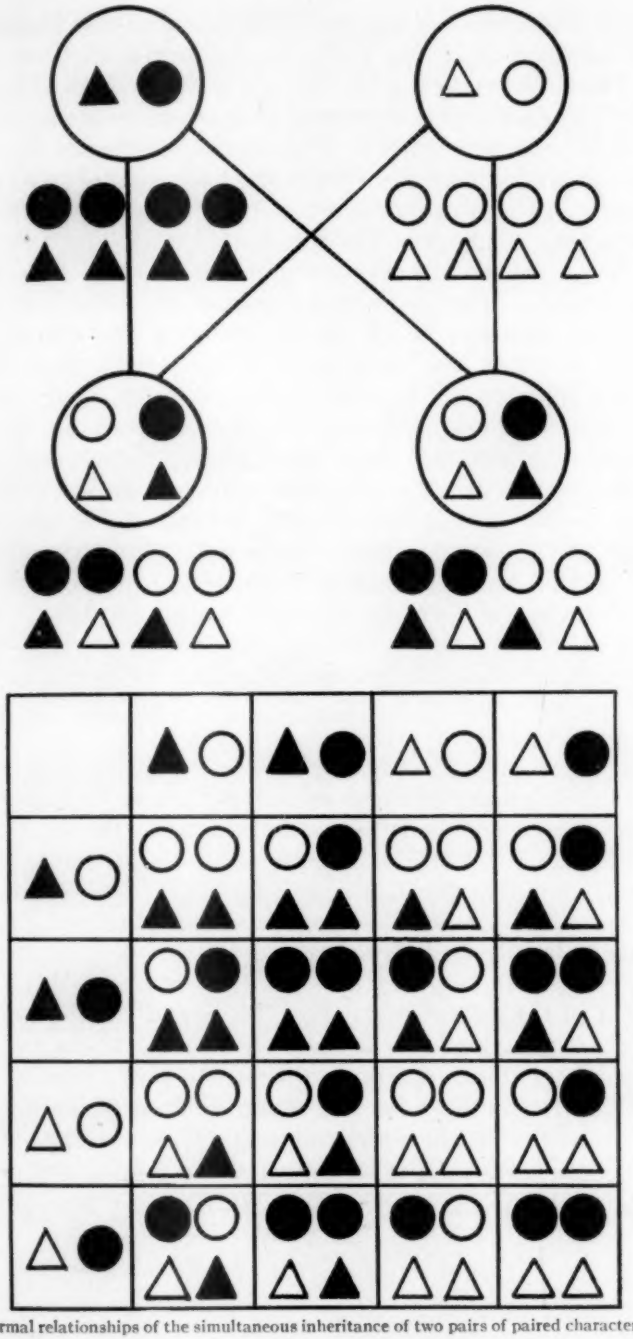
chart. The fowl to the left is the pure dominant black; the one to the right is the pure recessive white and the two central ones are mixtures toward the dark dominant color but with the characteristically blue admixture.

FIG. 2.



The genetic relationships of two paired characteristics in the blue Andalusian fowl.

FIG. 3.



The formal relationships of the simultaneous inheritance of two pairs of paired characteristics.

If one considers the multiple inheritance of two characters simultaneously again the principles of probability and pure chance may be applied. In Fig. 3 are shown symbols representing dominant and recessive characters of each of two inheritable characteristics.[5] This again represents the pure chance situation. At the top are the two original pure strain parents. One pair of characters is represented by circles and the other by triangles. The left hand parent is all black—both circles and triangles of the gametes directly underneath are all black. The right hand parent is all white—both circles and triangles of all of the gametes are white. The crossing of these two pure strain parents will give the F_1 offspring represented by the middle row of large circles, whose admixture is indicated by both containing all four symbols—a white and black circle and a white and black triangle. And the four gametes directly under each are exactly similar groups—to the left, a black circle and black triangle to the right, a white circle and white triangle and in the middle, the two possible mixtures of a black circle and white triangle and a white circle with a black triangle. These are the only four possible combinations which could be found among the gametes. But any of the four gametes from one hybrid parent could mix with any of the four gametes of the other hybrid parent.

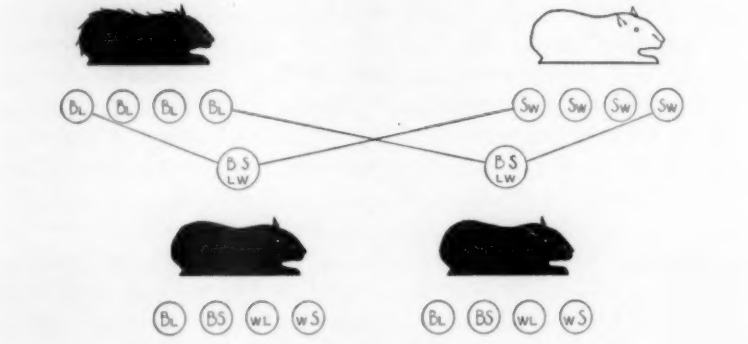
The lower part of the chart represents all possible combinations of mixtures of these mixed gametes. Consider that the four gametes of the left hand hybrid are in the left column of the square and the four gametes of the right hand hybrid are represented in the top row. The other 16 squares will then represent all possible combinations of these complex gametes. It will be observed that only two squares represent the original pure types of the grandparents—the third from the right in the second row is pure black and the second from the right in the third row is pure white.

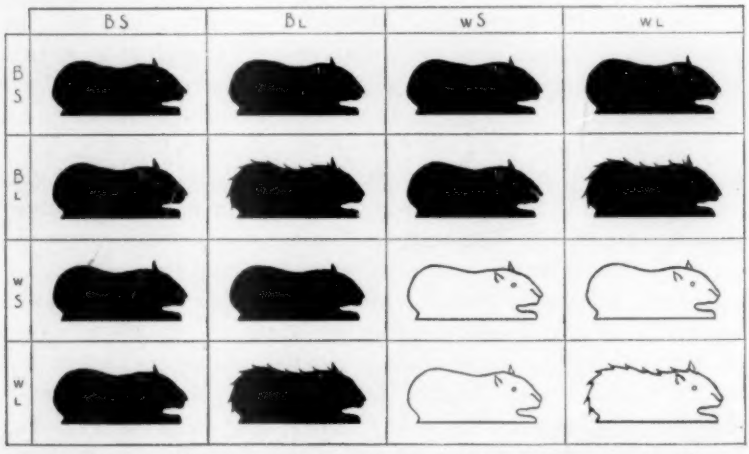
This chart again represents theoretical pure chance. The actual genetic findings for this situation in a particular case are represented in the fourth figure. These are the experimental findings of the inheritance in the guinea pig of hair

[5] Adapted from Hance, R. T., "Mendelism for the Blind," *J. Heredity*, 1936, 27. 151–153.

color and hair texture simultaneously. Of the two parent pure strain animals, the left one is black with long hair and the right one is white with smooth hair. Directly under each of these animals in the top row are indicated the fact that all

FIG. 4.





The genetic relationships of hair color and hair texture simultaneously in the guinea pig.

of the gametes of each parent are similar. Their offspring F_1 are the mixed hybrids represented in the middle row of animals on the chart but both are black with smooth coat because black and smooth coat are both dominant in this case. The dominant appears in both cases of this first

generation of hybrids even though one characteristic is obtained from each of the two parents. But the gametes of these hybrids represent all four possible combinations of these two characteristics.

These are again represented in the left hand column and the top row of the square at the bottom of the chart by the letters B and W for black and white and L and S for long and smooth coat. In the other 16 squares are represented the sort of animals found experimentally for these different combinations of gametes. Of these 16 cases, only 4 animals are of the recessive white color—the four cases in the lower right hand corner—and all of the rest are at least approaching the dominant black in color. Of the total 16, only 4 animals have the recessive long hair (those in the second and last columns or rows 2 and 4) and all of the rest have the dominant smooth coat. The second animal in the second row is the only one representing the original long haired black parent and the third animal in the third row is the only one representing the smooth coated white parental stock.

Here ends essentially Mendel's own work. His predecessors had been primarily interested in producing hybrids. Mendel having first recognized that characters might be considered as paired, made the distinction between dominant and recessive and then, by careful planning of his experiments and by careful observation and the application of statistical methods, worked out the *modes* of inheritance through several generations and determined that, after the second generation, we may expect these ratios to remain constant.

Much has been added to our knowledge of genetics since Mendel's time. Observations by Bateson [6] in 1894 indicated that variation in animals was discontinuous rather than slowly continuous. Hugo de Vries [7] in the first years of this century proved experimentally that sudden variations occurred in the primrose and to these sudden discontinuous changes he gave the name *mutations*. It has recently been shown that treating specimens with x-ray will produce these mutations more rapidly than their normal occurrence in

[6] W. Bateson, "Materials for the Study of Variation," 1894.

[7] H. de Vries, "Die Mutationstheorie." 2 vols. Leipzig. 1901–1903, Veit & Co.

nature. The geneticist can now produce mutations for study without patiently or impatiently waiting for their normal occurrence. McClung [8] was the first to insist that the *chromosome*—tiny dark-staining bodies visible in the nucleus of the cell at the time of cell division—was the important carrier of the hereditary characteristic. Further study along these lines has led to the development of the theory of the genes, considered the unit of inheritance and transmitted in the germ cells. It is believed that the genes are arranged linearly in the chromosomes and hence a gene from each parent cell lying next to or opposite one from the other parent cell will combine to determine this special characteristic in the offspring in the purely chance manner required by the Mendelian ratio. Many recent experiments indicate a degree of complexity in the inheritance of certain characteristics which makes their discussion at the present time and place not only impossible but, at best, certainly unprofitable. The genetic literature now abounds with descriptions of the modes of inheritance of many characteristics of a multitude of different plants and animals.

And now, what of the man who started this systematic study of genetic problems which has developed to such an extent that it may be considered a new science and from which one may expect so much in the way of scientific and of practical results in the future. I think that I shall break an all-time precedent by beginning the outline of a man's life [9] by reading his death notice written by himself several days before he died. It reads:

"The Augustinian Monastery of St. Thomas at Altbrünn in Moravia respectfully and with profound regret informs the public of the death of the Right Reverend Abbot

Gregor Joh. Mendel

Mitred Prelate, Companion of the Royal and Imperial Order of Francis Joseph, emeritus chairman of the Moravian Mortgage Bank, member and one of the founders of the

[8] C. E. McClung, "The Accessory Chromosome—Sex Determinant?" *Biol. Bull.*, 1902, **3**, p. 43.

[9] Probably the best and most available life of Mendel is by H. Iltis, "Life of Mendel" (Translated by E. & C. Paul), 1932. N. Y.: Norton. Pp. 336.

Austrian Meteorological Society, member of the Royal and Imperial Moravian and Silesian Agricultural Society, and various other learned and useful organizations, etc., etc. Born in Heinzendorf in Eastern Silesia on July 22, 1822. After a long, severe and painful illness, having received the holy sacrament and having submitted himself to the will of the Most High, he departed this life at half past one in the morning of January 6.

"The funeral ceremony will take place at the Monastery church on January 9th at nine in the morning, and thereafter the body of the deceased will be conveyed to the Brünn Central Cemetery for the final rest.

R.I.P.

Brünn, Monastery of St. Thomas.
January 6, 1884."

This is a man's own estimate of his life which, at least, mentions those happenings by which he, apparently, wished to be remembered. During his sixty-two years, the only things which Mendel himself apparently wished recorded are the facts that he was an abbot with the rank of Bishop, that he received a decoration from the Emperor, that he was a successful business man and that he was a member of a number of societies—only two of which are mentioned. To me, the outstanding thing in this self-estimate of Mendel's life is the complete omission of all mention of his experiments carried out so thoroughly, so successfully and so painstakingly over a fifteen-year period from 1856 to 1871. The reasons for this omission will become apparent when one considers the history of his publications.

Mendel was born in 1822, of peasant stock, in a small town in the corner of Silesia close to where the present German, Polish and Czechoslovakian borders come together. His father was interested in farming and particularly in fruit growing, so that the young Mendel had very early experience with practical agriculture. But the father increased his orchards to such an extent that he found himself short of capital and without the ready cash to properly provide for his son's education.

The young Johann Mendel—the Gregor was his monastic

name adopted later in life—was sent to a primary school in a nearby town but, due to his father's straightened financial circumstances, the boy was entered with only half rations—the parents sending supplementary food whenever possible. The result was that his primary school years were interrupted with frequent and lengthy illnesses. At the age of 18, however, he entered the High School of another town nearby at Olmütz, hoping to get some tutoring and so pay his expenses. In this he failed for the first year but acquiring some pupils the second year, he continued his studies and completed the course of study.

After graduation, it became apparent to young Mendel that there was no future for him in the educational field and so he sought recommendation to the St. Thomas Cloister at Brünn, where he was admitted as a novice in 1843. In many ways, this was a fortunate appointment for Mendel. This particular order had traditional interests in the arts and sciences. Furthermore, Prelate Napp, who was Abbot at the time, was particularly interested in natural philosophy and especially in botany and he endorsed Mendel's desire to study and experiment to the utmost. This association with Abbot Napp during his early monastic years is one of the few "breaks" which Mendel had during his lifetime.

In 1845, he began a four-year course at the Brünn Theological College to prepare himself for the priesthood. Another "break" occurred two years later. An epidemic in Brünn had caused many deaths and the Monastery found itself short handed. In the face of this situation, Abbot Napp asked that Mendel be given higher orders by special dispensation and, this being granted, he was made sub-deacon, deacon and priest within a three week period. In 1848, Mendel began his duties as parish priest and within a little over a year it became evident that he was not temperamentally suited to this kind of a life. One of his duties as parish priest was to visit the sick and the dying and such visits invariably made him physically ill. As a result, he was relieved of his duties as parish priest and he became a substitute teacher at the Znaim High School, in a small town near Brünn. Here he taught all branches of Natural Science apparently with success during the academic year of 1849–1850.

In the latter year, Mendel showed particularly bad judgment in attempting his examination for a State Certificate as a high school teacher in the Natural Sciences. With only a high school education himself and with no university training, with only the background of a year's teaching experience and with little time to prepare himself for the examinations, his complete failure could have been predicted. There is a record of that examination and its study indicates that candidates who were successful in passing really earned their teaching certificates. He was first given the task of writing an essay in physics, one in geology and one in natural history. None of the written examinations were considered satisfactory and most of the oral examinations which followed were even less so. One of the examiners, a Professor Kner, sent the following report which would seem to embody a highly sympathetic realization of Mendel's situation and of his difficulties. "The result of the oral examination in natural history was more satisfactory to this extent, that the candidate showed more knowledge and gave evidence of more diligent study than might have been expected from the written papers. It was plain that he was devoid neither of industry nor of talent. It would seem, however, that he can have had no opportunity for acquiring exhaustive knowledge, and must have lacked access to the necessary means of study, so that he is not as yet competent to become a teacher. Still we may hope that if he is given opportunity for more exhaustive study together with access to better sources of information he will soon be able to fit himself, at least for work as a teacher in the Lower schools."

Abbot Napp took this advice sympathetically and seriously and sent Mendel—now almost 30 years old—to the University of Vienna for study from 1851–1853. The next year he was appointed as substitute teacher in physics and natural history in the newly formed Brünn Modern School where he remained until 1868. In 1856, he again tried the examination for state certification and, in spite of his increased preparation and teaching experience, he again completely failed. Mendel thus taught for nearly 20 years and never ranked better than substitute teacher.

One incident of these years seems worth recording inas-

much as it indicates something of Mendel's attitude and methods as a teacher. It has been my privilege to meet one of the few, if not the only surviving pupil of Mendel while he was a teacher at the Brünn Modern School—an old gentleman still living in Brünn. I have mentioned that the Monastery buildings faced the hill called the Spielberg. Mendel's rooms were on that side of the monastery building. It was the custom of the pupils to wander about the slopes of the Spielberg of an evening where they would chat and play and where there was always a chance that some girls might also be out for a stroll. This seemed a waste of time to the teacher Mendel inasmuch as it took them away from the more serious duties of study. So Mendel made a practice of mounting his telescope in his bedroom window and of training it on the slopes of the Spielberg rather than on the heavens, in the hope of catching some of his errant pupils. And if he succeeded, that pupil could expect to be called for a full hour's recitation the next day.

After failing the certification examination a second time, Mendel apparently gave up hope of ever succeeding. With less urge to study and now well established in his teaching, he found more leisure and this leisure he utilized in his botanical experiments which he began in 1856—the same year as the second examination failure. The first series of experiments—the outstanding results of which I have already outlined—were completed and led to publication in 1866. During part of this time, Mendel was in correspondence with an academic botanist, Professor Carl von Nägeli, who in 1867 induced him to switch his experimental material from the edible pea to the hawkweed. Experiments with this new material continued until 1871, but these last four years work led to unfortunate failure inasmuch as the hawkweed proved as unsuitable a type of plant for this sort of experiment as the edible pea had been highly satisfactory. The results of some of his work with the hawkweed were published in 1869.

During all of this time, Mendel was also interested in climatology and for many years he ran one of the stations for meteorological research in Moravia. He was interested in the sub-soil water level, in sunspots and tornados—and on the two latter subjects he published papers.

The year 1868 is a turning point in the career of Mendel. At this time he was elected Abbot and Prelate of the St. Thomas Monastery. There are letters which indicate that he was anxious to obtain this election primarily because he hoped that he would have more time for his experiments. The election, of course, entailed his resignation as a teacher. The election did just the opposite of what Mendel had hoped. From an obscure substitute teacher, Mendel suddenly found himself immersed in business. There was not only an increased call for ecclesiastical functions but the Monastery was wealthy and owned many outlying farms and a very large dairy farm, all outside of Brünn. The result was that he found himself much busier than before and besides he had to be away from Brünn more and more frequently and for longer and longer periods of time. He also had to attend to much communal work of one sort or another such as becoming Chairman of the Moravian Mortgage Bank. The result was that no one took proper care of his experimental plants during his absences and, in 1871—only three years after his election as Abbot—he gave up his research completely.

Mendel's later years were harassed with a fight regarding the taxation of church property—a very serious thing indeed for a wealthy Monastery. In 1874 a bill was passed taxing church property, the income to be paid into a religious fund. The next year the government required the St. Thomas Monastery to make a tax return with which Mendel complied. But when a tax was assessed on the basis of this return, Mendel protested and he continued to protest and refused to pay the tax until his death. He was ill for almost a year—the illness being partly the result of the worry of the tax fight with the government but due partly to arterial and cardiac degeneration due to excessive fat aggravated by excessively heavy smoking. Mendel apparently smoked about twenty cigars a day during his later years and anyone who has tried to smoke Austrian tobacco will realize what this must have meant.

Mendel's life is thus a curious mixture of success and of failure. His election as mitered abbot raised him high above the average level attained by his peers. But from another

point of view his life was a series of failures. One can point to the frequent interruptions of his early education, to his entering the priesthood because the desired educational career did not promise success and then to his failure as a parish priest, to his twice failing his certification examinations and being forced to continue for nearly twenty years with the rank of a substitute teacher, to his switching from the suitable pea to the unsuitable hawkweed as experimental material, to his election as Abbot which so hastened the close of his research period and to the tax litigation with the government which certainly hastened his end.

This makes a sorry list of failures indeed—a list of the thwarting of life's ambitions. Nor did the failures end with his passing. Shortly after Mendel's death, his successor effected a compromise with the government in the tax matter and so undid the results of Mendel's ten-year fight which had been successful. And again, some years ago a statue of Mendel was erected in Brünn in memory of him and his work. It is a dignified statue showing him in his monastic robes. But shortly after the war, during some anti-religious riots, the nose was broken from the statue. So Mendel stands today facing his monastery buildings still in this mutilated condition.

But the most tragic failure of all, in the life of Mendel, has been left for the last. This has to do with the reception given by the scientific world to his epoch-making publication on the inheritance of the edible pea. In 1862, Mendel was one of the founders of a local club for the discussion of scientific subjects called The Brünn Society for the Study of Natural Science. Mendel read his paper entitled "Experiments in Plant-Hybridization" to two successive meetings of the Society in February and March, 1865, and it was printed in Volume 4 of their Proceedings which appeared in 1866. The reports of the meeting show that there was no discussion of the paper—no one seemed to understand the results and certainly no one appreciated their significance. The records of the Society show that Mendel received 40 offprints for private distribution and that the Society distributed 120 copies to other societies, academies and universities in Austria and abroad. Of a possible 160 copies distributed, no one

noticed the paper. It received no criticism and no comment although certainly Professor Nägeli and various other professional botanists must have seen it.[10]

The paper went unnoticed during Mendel's lifetime with the single exception of a very casual mention by Focke [11] in a book published in 1881. One has here another example of how a paper may be completely buried, no matter what its significance and value, unless very shortly after its publication it is taken up and discussed in the literature of the subject. And this is especially true of the paper published in an obscure out-of-the-way publication such as the Transactions of the Brünn Society for the Study of Natural Science.

With the exception of this single quotation, the paper remained buried for thirty-four years—from its publication in 1866 to its rediscovery in 1900. But it was through this single reference that the Mendel papers were rediscovered. In 1900 three workers appeared to have discovered these articles almost simultaneously. They were Hugo de Vries, E. Tschermak and C. Correns.[12] A translation of the two papers into English was immediately prepared by C. T. Druery and printed in 1901 in the *Journal of the Royal Horticultural Society*. In the next year, W. Bateson published a separate translation under the title of "Mendel's Principles of Heredity: A Defence." This book has gone through a number of editions and reprintings and is today the most readily available source of the original papers. In the light of our increased knowledge of genetics at the time of the rediscovery, the importance of these early papers was immediately recognized. And during these last thirty-five years, Mendel's Law has become almost a household term. It is to the ever-

[10] There are a few extant copies of the original paper. The best source for reading the original is the translation found in W. Bateson, "Mendel's Principles of Heredity," 1930, Cambridge: University Press. In this source the paper on the edible pea is found pp. 335–379 and that on the hawkweed—the 1869 paper—pp. 380–386.

[11] W. O. Focke, "Die Pflanzenmischlinge," 1881, p. 109.

[12] H. de Vries, "Sur la loi de disjonction des hybrids," *C. R. Acad. Sci.*, 1900, 130; E. Tschermak, "Ueber künstliche Kreuzung bei Pisum sativum," *Zeits. f. d. landw. Versuchsw. in Oesterr.*, 1900, **3**, Heft 5; C. Correns, "G. Mendel's Regal über das Verhalten der Nachkommenschaft der Rassenbastarde," *Ber. Deutsch. Bot. Gesellsch.*, 1900, **17**, 158ff.

lasting credit of his rediscoverers that they gave credit where it was due and labeled this concept "Mendel's Law" in honor of its real discoverer. This is more especially true because both de Vries and Correns had completed experiments leading to similar conclusions before they had discovered the existence of the Mendel papers. Correns writes: "In noting the regular succession of the phenomena (of hybridization) and in finding an explanation for them, I believed myself, as de Vries believes himself, to be an innovator. Subsequently however, I found that in Brünn during the 'sixties Abbot Gregor Mendel, devoting many years to the most extensive experiments on peas, had not only obtained the same results as de Vries and myself, but had actually given the very same explanation, so far as this was possible in the year 1866. . . . This paper of Mendel's, to which Focke refers (though without full justice to its importance) in his "Pflanzmischlinge," is among the best works ever written upon the subject of hybrids." This is high praise indeed for a colleague who one finds has more or less stolen one's thunder!

There is one other tragedy in connection with Mendel's writings—namely, that they never got into the hands of Charles Darwin. Darwin conceived of evolution as a slow gradual process which in time led to the differentiation of species. It is a popular misconception that the theories of Darwin and of Lamarck are completely opposed to each other. Rather Darwin himself seemed to lean on Lamarckian principles from time to time as an explanation of variation which was one of the important principles of his evolutionary theory. True the "Origin of Species" and the "Fertilization of Orchids" were published before the Mendel papers. But had Darwin seen Mendel's work and had he understood it, the ideas of dominance and the mechanism of inheritance therein contained, there can be no doubt that the "Variation of Animals and Plants Under Domestication," "The Descent of Man" and "The Expression of Emotions" would have appeared in different form. But Darwin could write only on the basis of his knowledge and his ideas were extended and popularized by the writings of such men as Huxley and Galton. These ideas remained current until they were modified in scientific thought by the rediscovery of the experiments of Mendel.

A few years ago a picture of Gregor Mendel was shown in a shop window in Brünn. Two townspeople stopped to look at it and one asked the other whose picture it was. The other replied: "Oh, that is Abbot Mendel, the man who increased our taxes by fighting the religious fund." In a word, this incident symbolizes the tragedy of Mendel's life. In spite of his election as Prelate, he believed himself a failure in almost everything that he attempted. One can understand that naturally he did not mention his experiments in his death notice—experiments whose publication had caused not a single comment, favorable or unfavorable, by his scientific colleagues. Probably most scientists would rather be imprisoned for fifteen years for their writings, as was Roger Bacon, than have them so completely ignored as were the writings of Mendel. Posthumous praise is poor consolation indeed!

As so often happens in any great scientific advance, Mendel's real contribution was essentially one of method. The development of the method of exact fertilization of the individual plant with the keeping of genealogies and the study of individual offspring and the statistical approach must lead to the results which he obtained.

In these days when huge sums are spent—and certainly most properly spent—upon research, it is interesting to note that, with only a small plot of unused garden at his disposal, with a few common garden tools at hand in any case and with a handful of dried peas; working alone and without advice of colleagues, but with infinite imagination and with the utmost care and persistence, Gregor Johann Mendel worked out one of the great laws to be found in the history of science.

ABSORPTION OF COSMIC RADIATION IN MATTER.

BY

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RESUMÉ.

The paper describes absorption and shower producing effects of cosmic radiation in water, coal, copper sulphate, iron sulphide, sand, and barium sulphate. The measurements were made with triple coincidence Geiger counters, near sea level. The coefficient of absorption was not found to be strictly proportional to density, substances of higher atomic number showing departures. The coefficients per bound electron was found to be constant, within the precision of the observations. Showers were found to follow the direction of the primaries, and to show maxima with increasing thicknesses of shower-producing matter in the region of 50–100 gms./cm.^{2} of material.

1. INTRODUCTION.

The investigation here reported consisted of measurement of absorption of cosmic radiation in six substances. Triple coincidence Geiger counters were used. The substances were water, sand, $CuSO_4$ snow, commercial FeS, barytes ($BaSO_4$), and coal. The experiments were carried on near sea level (65 meters) in the open (water and sand) or under a light wooden roof ($CuSO_4$, coal, FeS, and $BaSO_4$).

2. APPARATUS AND METHOD.

The absorbing materials were held in a wooden hopper and the counters were mounted in a vertical row underneath. The hopper was 105 cm. deep and of the form of an inverted truncated rectangular pyramid, so proportioned that no ionizing ray could pass through all three counter tubes without traversing the full depth of the hopper. In the case of the $BaSO_4$ the total amount in the hopper was about 900 kg. The counters were connected to detecting, amplifying and recording circuits like those used by Johnson and Street.[1] Practically the only change in their circuits was the substitu-

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[1] T. H. Johnson and J. C. Street, FRANK. INST., J., 215, 239 (1933).

tion of a -45 tube in the third stage of the relay-recorder circuit in order to have a greater current for the operation of the relay. The recorder was a telephone message recorder.

FIG. 1.

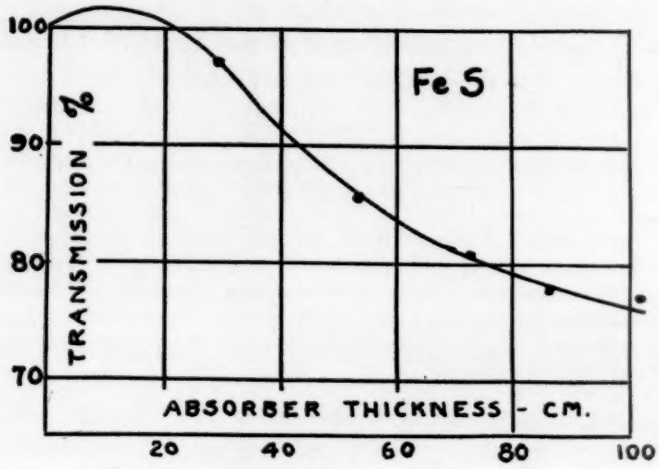
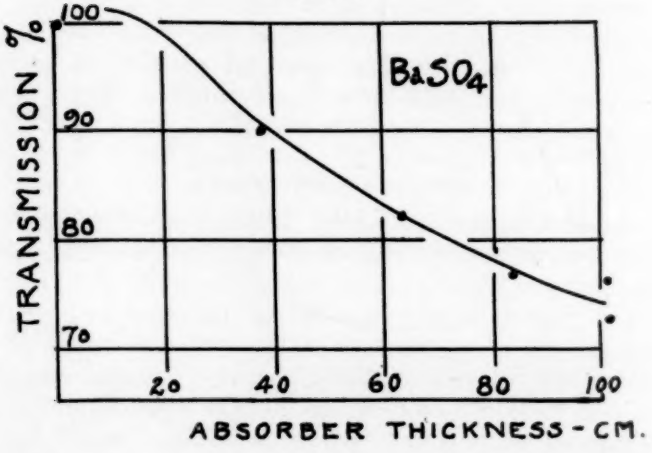


FIG. 2.



Dry cells were used for all purposes except filament supply. Mercury tube rectifiers were tried, for high voltage supply and for plate and bias purposes, but were abandoned because of power supply variations and disturbances, which apparently

caused many spurious counts. Very complete shielding and grounding were found to be necessary.

Arrangements were made for swinging the counters out from under the hopper so that zero-absorber runs could easily be alternated with absorber runs. Runs were made at not less than four thicknesses of absorber in each case. The usual length of run was 72 hours.

Tests for showers were made with the tubes out of line or otherwise disposed, as indicated in Figs. 4 and 5.

Transition effects and equilibrium appeared to be complete after a few centimeters of absorber, as indicated in the curves of Figs. 1 and 2. Radioactivity, inherent in the test substances, was found to have a negligible effect, as shown by tests made with counters out of line but located close to the test substances. Variations of atmospheric pressure were never great, the maximum being 768 mm. and the minimum, 751 mm. The greatest variation in any one week was 10 mm. and the usual was 4 mm. Temperature variations were not large. The high voltage battery was more or less heat insulated and its voltage did not vary appreciably, on this account. On some hot days a crude air conditioning equipment was employed to keep the counters at a satisfactory operating temperature. Effects of fluctuations of the above varieties tended to cancel out due to frequent zero-absorber runs (really calibrating runs).

3. ABSORPTION TESTS.

Graphs showing the results of transmission vs. absorber thickness for two of the test substances are shown in Figs. 1 and 2. The coefficient of absorption μ per centimeter was computed for each substance on the basis of an exponential relation and from these and the measured densities and the

TABLE 1.

	μ.	Bound Electrons per cc.	μ per Bound Electron.
H_2O	$.151\times10^{-2}$	3.35×10^{23}	45×10^{-25}
C	.166	3.20	52
$CuSO_4$	.219	4.26	51
SiO_2	.256	4.84	53
FeS	.332	7.39	45
$BaSO_4$	.355	7.66	46

known compositions, values of μ per bound electron were calculated. These values are shown in Table 1. Figure 3 shows the results of μ per cm. vs. density, and indicates that the absorption was not strictly proportional to the density. Some of the density values are not so great as might be expected, due to loose packing of the granular materials.

FIG. 3.

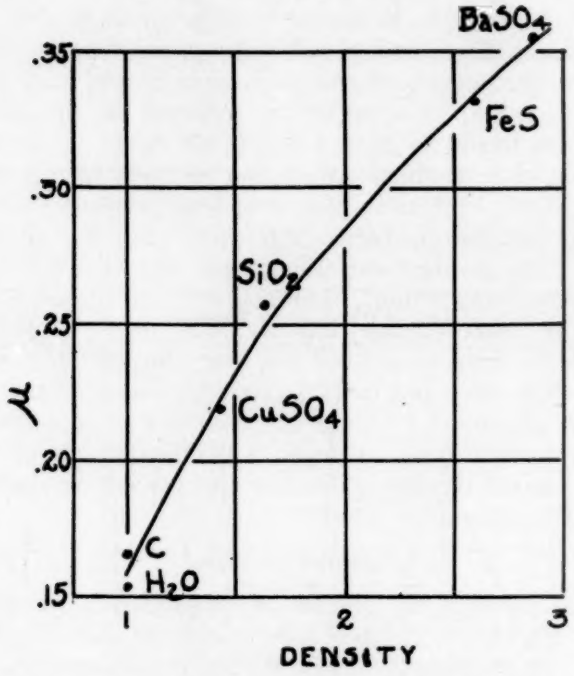


Table 1 indicates that within the precision of the experiments the value of μ per bound electron is constant.

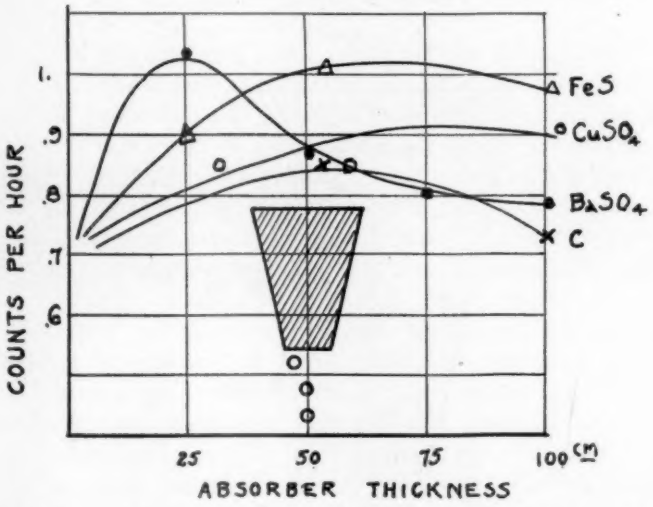
In the case of water, the ratio of cosmic ray intensity at 1 meter depth to that at sea level in air, as obtained by Millikan was .84 and in this investigation it was .86. His results were for all-sided incidence while these are for nearly vertical, only.

4. SHOWER TESTS.

When the top counter tube was offset as shown in Fig. 4, coincidences occurred only when shower action took place,

and the curves indicate how this action varied with the amount of scattering or shower-producing material which was present. In each case a maximum is indicated as was discovered to be true by Rossi [2] who found the counts to increase up to a thickness of about 20 gms./cm.^{2} for Pb, and up to a thickness of about 60 gms./cm.^{2} for Fe, and then to fall off due to the absorption of excited secondaries. The present results

FIG. 4.



show the maximum to occur at 72 gms./cm.^{2} for $BaSO_4$, 135 gms./cm.^{2} for FeS, 112 gms./cm.^{2} for $CuSO_4$, and 60 gms./cm.^{2} for C.

In view of the known low penetrability of showers, these results suggest that shower producing radiation reaches equilibrium with primaries after passing through 50–100 gms./cm.^{2} of material. The observation that the optimum thicknesses for the production of showers vary in no regular way with atomic number suggests that the showers are due to specific nuclear properties. Further studies of showers in this way may, therefore, reveal significant nuclear information.

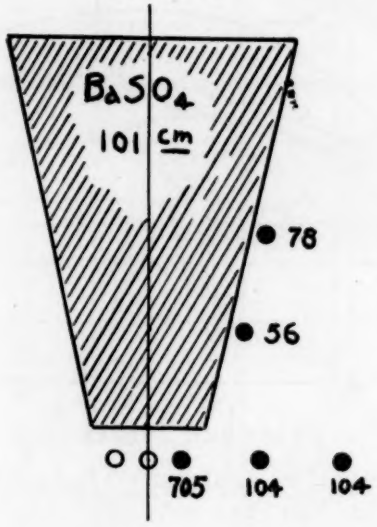
Other tests made with the same arrangement, except that the off-set distance d of the top tube was made variable and

[2] B. Rossi, *Zt. f. Phys.*, 82, 151 (1933).

the thickness constant at 101.5 cm., showed rapid decrease of counts as d increased, showing that the spraying action was confined to a small angle and that the secondaries followed generally in the same direction as their primaries. The rate of decrease was approximately the same for the four materials. In all the above tests, more counts were recorded when the absorber was present than when it was not.

Figure 5 shows, approximately to scale, other test arrangements. The two counter tubes shown by open circles were

FIG. 5.



held in a fixed position while the third was moved successively to the various positions shown by the full black circles. The numbers show the number of counts recorded in the same length of time for each position. These results indicate that the secondaries produced in the upper layers did not often reach the counter tubes, and that, again, the secondaries follow closely in the same direction as their primaries.

The author wishes to thank Professor E. O. Lawrence, who suggested the problem, for his helpful advice on numerous occasions, and for his continued interest in the work as it progressed.

THE DESIGN OF A MECHANICAL ANALOGY FOR THE GENERAL LINEAR ELECTRICAL NETWORK WITH LUMPED PARAMETERS.

BY

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Within the last few years great strides have been made in communication engineering and the analysis of complicated electrical networks has become well organized. There does not seem, however, to be a corresponding systematic treatment of mechanical vibration systems analogous to complicated electrical networks. Although mechanical analogies to specific electrical networks have been given, as far as we are aware there has been no mechanical analogy described for the general linear electrical network. Exhaustive theoretical treatments of mechanical vibration systems have been given [1] which are in general too complex for the average engineer.

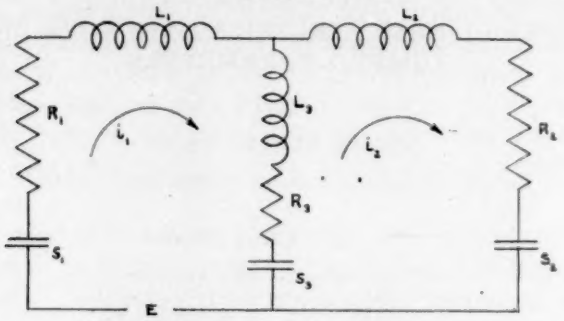
In the following description of a mechanical analogy to the general linear electrical network an attempt is made to systematize and simplify the analysis of certain mechanical vibration systems so that the engineer familiar with the elements of electrical network theory may readily recognize and analyze mechanical systems analogous to electrical networks, or conversely, set up mechanical systems analogous to given electrical networks. The electrical engineer concerned with acoustical, seismic, or other mechanical vibration problems needs to bridge the gap to the mechanical vibration field where existing analytical methods are not as highly organized as in his own field.

Some commonly used methods for electrical network analysis are not suitable for analysis of the corresponding mechanical system. A method is here devised which, while not fundamentally new, involves a new line of reasoning well adapted to the analysis of either the electrical network or of

[1] Lord Rayleigh, "Theory of Sound," Vol. I.

its corresponding mechanical system. The method is applied to the two mesh electrical network of Fig. 1 and to the bevel

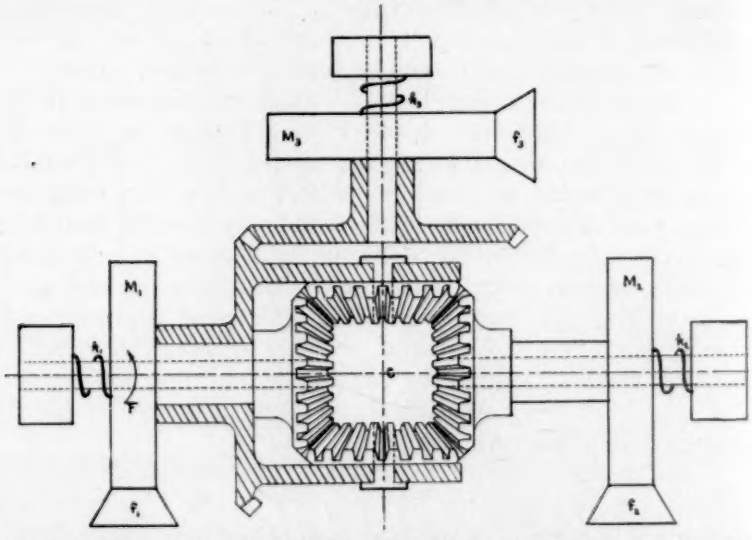
FIG. 1.



An electrical network with two degrees of freedom.

gear differential with its associated springs k, damping vanes f, and flywheels M, shown in Fig. 2. This mechanical

FIG. 2.



A bevel gear differential with associated flywheels, springs, and damping vanes: a mechanical analogue for the electrical network of Fig. 1.

system will be shown to be analogous to the electrical network of Fig. 1. With the removal of certain of the springs, damping

vanes, or flywheels, the system will be shown to be analogous to any two mesh electrical network with constant parameters. A pendulum model involving a differential gear and which is analogous to an inductively coupled two mesh electrical network has been described [2] but the device described in this paper is more flexible in that it is analogous to *any* linear two mesh electrical network. Also, as will be shown, this new device may be combined with other similar units to form mechanical systems analogous to complicated electrical networks. Subsequently the aforementioned method is extended to the general linear electrical network and to a corresponding mechanical system of n degrees of freedom involving combinations of bevel gear differentials. Several examples are illustrated and analyzed. In the appendix the bevel gear differential of Fig. 2 is analyzed by the conventional, though more complicated, method of Lagrange.

In all cases the analogies will consist in the identity of form of the differential equations involved. The torque applied to one of the flywheels of the mechanical system will correspond to the e.m.f. applied to a mesh of the electrical network and the angular velocities of the several flywheels will correspond to the currents in the meshes of the analogous electrical network.

In the analysis of electrical networks we shall consider that each mesh of the network contains inductance, resistance, and capacitance on its contour and, likewise, that each mutual branch contains all three kinds of elements making the coupling between any two meshes of the most general type. Each mesh will be assumed to be coupled to every other mesh in the network by means of inductance, resistance and capacitance.

Figure 1 representing a two mesh electrical network includes this general type of coupling in the common branch together with additional inductance, resistance, and capacitance in each mesh. This is an electrical network of two degrees of freedom in which the mesh currents i_1 and i_2 may be considered as the independent "coördinates." We may

[2] L. Mandelstam, "Pendelmodell zur Demonstration Electrischer Gekoppelter Kreise," *Jahrbuch der Drahtlosen Telegraphie und Telephonie*, Vol. 4, p. 515, 1911.

derive the differential equations for this network by summing up the voltages and counter voltages in each mesh. This is the conventional method but here we will modify this process with a new line of reasoning which will be seen to be peculiarly adapted to the analysis of the corresponding mechanical system.

Considering mesh No. 2 of Fig. 1 as open circuited (hence $i_2 = 0$) and summing up the counter voltages in mesh No. 1 we obtain

$$(L_1 + L_3)\frac{di_1}{dt} + (R_1 + R_3)i_1 + (S_1 + S_3)\int i_1 dt,$$

where L, R, and S represent inductance, resistance, and elastance $(1/C)$ respectively. This is the counter voltage produced in mesh No. 1 by the current i_1, and may be written $a_{11}i_1$, where a_{11} is the differential-integral operator [3]

$$\left\{ (L_1 + L_3)\frac{d}{dt} + (R_1 + R_3) + (S_1 + S_3)\int dt \right\}.$$

In addition to this there is a counter voltage produced in mesh No. 1 by the current i_2 which may be found by considering mesh No. 1 as open circuited (hence $i_1 = 0$), and summing up the counter voltages in mesh No. 1 produced by the current i_2. The summation gives

$$-L_3\frac{di_2}{dt} - R_3 i_2 - S_3\int i_2 dt,$$

which may be written $a_{12}i_1$, where a_{12} is the differential-integral operator

$$\left\{ -L_3\frac{d}{dt} - R_3 - S_3\int dt \right\}.$$

The terms are negative because the assumed positive direction for the current i_2 through the common branch is opposite to that for the current i_1 in this branch. The total counter voltage developed in mesh No. 1 may now be set equal to the applied voltage E giving

$$a_{11}i_1 + a_{12}i_2 = E.$$

[3] Lord Rayleigh, loc. cit., p. 104.

Similarly we may obtain the counter voltage produced in mesh No. 2 by current i_2 by considering mesh No. 1 as open-circuited ($i_1 = 0$) and summing up the counter voltages in mesh No. 2. This may be written $a_{22}i_2$, where a_{22} is the differential-integral operator

$$\left\{ (L_2 + L_3)\frac{d}{dt} + (R_2 + R_3) + (S_2 + S_3)\int dt \right\}.$$

The counter voltage produced in mesh No. 2 by the current i_1 may be obtained by considering mesh No. 2 as open-circuited ($i_2 = 0$) and summing up the counter voltages in mesh No. 2 produced by the current i_1. This summation may be written $a_{21}i_1$, where a_{21} is the differential-integral operator

$$\left\{ -L_3\frac{d}{dt} - R_3 - S_3\int dt \right\}.$$

Note that the operators a_{21} and a_{12} are equal. The total counter voltage developed in mesh No. 2 set equal to zero gives

$$a_{21}i_1 + a_{22}i_2 = 0.$$

The differential equations for the two mesh electrical network of Fig. 1 may therefore be written

$$\begin{aligned} a_{11}i_1 + a_{12}i_2 &= E, \\ a_{21}i_1 + a_{22}i_2 &= 0. \end{aligned} \qquad (1)$$

As will be shown later these equations are of the same form as the equations of motion for the bevel gear differential of Fig. 2 which will be seen to be analogous to the two mesh electrical network of Fig. 1.

In the above analysis of the two-mesh electrical network the process of considering one mesh as open-circuited while summing up counter voltages may seem superfluous because the process is tacitly assumed in the conventional application of Kirchoff's laws to the meshes of an electrical network but the detailed reasoning in this process is particularly helpful in the analysis of the corresponding mechanical system. This will now be demonstrated by application to the bevel gear differential of Fig. 2. The method, while leading to the same result, will be seen to be simpler and more direct than

the conventional use of Lagrange's equations as given in the appendix. This is particularly true in the application to more complicated mechanical systems. The method should be compared step by step with its application to the analogous two mesh electrical network as given above. It will be observed that the reasoning is the same as that for the electrical network if we make the word substitutions tabulated below:

In the Electrical Network	*In the Mechanical System*
mesh	flywheel
open-circuited	clamped
i_2, i_1	ω_2, ω_1
voltages	torques
inductance L	moment of inertia K
resistance R	damping constant f
elastance S	spring constant k

The bevel gear differential as shown in Fig. 2 is similar to that commonly used in automobiles, the difference being that in this differential the gear ratios have been altered. M_1 and M_2 are flywheels mounted on shafts corresponding to those of the rear wheels of an automobile and the flywheel M_3 is mounted on the shaft corresponding to the drive shaft. Each flywheel is attached to a spiral spring k (not helical as shown) which provides a restoring torque when any of the flywheels is disturbed. The flywheels are provided also with damping vanes indicated schematically as f. The arrangement of the gears and the gear ratios is such that the following relative motions of the flywheels occur if we consider all flywheel rotations as positive when counter-clockwise as viewed towards the center C. With M_3 clamped a positive rotation of M_1 with angular velocity ω_1 causes a positive rotation of M_2 with angular velocity ω_1. With M_2 clamped a positive rotation of M_1 with angular velocity ω_1 gives a negative rotation of M_3 with angular velocity ω_1. With M_1 clamped and positive rotation of M_2 with angular velocity ω_2, M_3 will rotate in the positive direction with angular velocity ω_2. If M_2 and M_1 both rotate positively with angular velocities respectively ω_2 and ω_1, M_3 will rotate with angular

velocity $(\omega_2 - \omega_1)$, in the positive direction if $\omega_2 > \omega_1$, in the negative direction if $\omega_2 < \omega_1$, and not at all if $\omega_2 = \omega_1$.

In the analysis to follow the springs k will be assumed to obey Hooke's law, damping resistances will be assumed proportional to the angular velocities, and the moments of inertia of the flywheels will be assumed to be much greater than those of their associated shafts and gears. An harmonic torque F is assumed applied to the flywheel M_1. With small displacements, electromagnetic damping which is proportional to the angular velocity, and relatively heavy flywheels these assumptions will be justified. The harmonic torque may be applied by means of a small motor with unbalanced flywheel mounted directly upon the large flywheel M_1.

Qualitatively it may be seen that as far as the motions of the flywheels are concerned M_1 is coupled to M_2 by means of M_3 together with its spring k_3 and damping vane f_3. The arrangement constitutes a coupled mechanical system of two degrees of freedom in which the independent coördinates ω_1 and ω_2, the angular velocities of M_1 and M_2 respectively, may be used to specify the flywheel velocities.

Considering flywheel No. 2 clamped (hence $\omega_2 = 0$) and summing up the counter torques on flywheel No. 1 we obtain

$$(K_1 + K_3)\frac{d\omega_1}{dt} + (f_1 + f_3)\omega_1 + (k_1 + k_3)\int \omega_1 dt,$$

where K, f, and k represent flywheel moments of inertia, damping constants, and spring constants respectively. This is the counter torque produced in flywheel No. 1 by its angular velocity ω_1 and may be written $a_{11}\omega_1$, where a_{11} is the differential-integral operator

$$\left\{(K_1 + K_3)\frac{d}{dt} + (f_1 + f_3) + (k_1 + k_3)\int dt\right\}.$$

In addition to this there is a counter torque on flywheel No. 1 produced by the angular velocity ω_2 of flywheel No. 2 which may be found by considering flywheel No. 1 as clamped (hence $\omega_1 = 0$) and summing up the counter torques on flywheel No. 1 produced by the angular velocity ω_2 of flywheel

No. 2. The summation gives

$$-K_3\frac{d\omega_2}{dt} - f_3\omega_2 - k_3\int\omega_2 dt,$$

which may be written $a_{12}\omega_2$, where a_{12} is the differential-integral operator

$$\left\{-K_3\frac{d}{dt} - f_3 - k_3\int dt\right\}.$$

The terms are negative because the assumed positive angular velocity ω_2 of flywheel No. 2 produces a counter torque on flywheel No. 1 which is opposite in direction from the counter torque on flywheel No. 1 produced by its own positive angular velocity ω_1. The total counter torque acting on flywheel No. 1 may now be set equal to the applied torque F giving

$$a_{11}\omega_1 + a_{12}\omega_2 = F.$$

Similarly we may obtain the counter torque produced in flywheel No. 2 by the angular velocity ω_2 of flywheel No. 2 by considering flywheel No. 1 as clamped ($\omega_1 = 0$) and summing up counter torques acting on flywheel No. 2. This may be written $a_{22}\omega_2$, where a_{22} is the differential-integral operator

$$\left\{(K_2 + K_3)\frac{d}{dt} + (f_2 + f_3) + (k_2 + k_3)\int dt\right\}.$$

The counter torque produced in flywheel No. 2 by the angular velocity ω_1 of flywheel No. 1 may be obtained by considering flywheel No. 2 as clamped ($\omega_2 = 0$) and summing up the counter torques on flywheel No. 2 produced by the angular velocity ω_1 of flywheel No. 1. This may be written $a_{21}\omega_1$, where a_{21} is the differential-integral operator

$$\left\{-K_3\frac{d}{dt} - f_3 - k_3\int dt\right\}.$$

Note that the operators a_{21} and a_{12} are equal. The total counter torque developed in flywheel No. 2 set equal to zero gives

$$a_{21}\omega_1 + a_{22}\omega_2 = 0.$$

The differential equations of motion for the bevel gear differential of Fig. 2 may therefore be written

$$\begin{aligned} a_{11}\omega_1 + a_{12}\omega_2 &= F, \\ a_{21}\omega_1 + a_{22}\omega_2 &= 0, \end{aligned} \tag{2}$$

which are of the same form as equations (1) for the two mesh electrical network of Fig. 1. This mechanical device is thus seen to be analogous to the two mesh electrical network, the torque F applied to flywheel M_1 corresponding to the voltage E applied to mesh No. 1, and the angular velocities ω_1 and ω_2 corresponding to the mesh currents i_1 and i_2 respectively.

In both the electrical and the mechanical systems the coefficients representing the differential-integral operators completely determine the behavior of the system. In either case these coefficients may readily be determined by inspection of the electrical network or mechanical system.

If the two mesh electrical network of Fig. 1 is altered by removal of any of its elements the analogous mechanical system may be derived from that of Fig. 2, by removal of the corresponding mechanical elements. It is thus possible to give the mechanical analogue to any two mesh electrical network. By clamping flywheel M_2 we have a mechanical system of one degree of freedom corresponding to a single mesh electrical network with inductance, resistance, and capacitance. With removal of one or more of the mechanical elements we have the analogue to the corresponding electrical network with one or more of its elements removed. When inductance is removed from a mesh of an electrical network the corresponding mechanical system will have a flywheel replaced by a light, relatively inertia-free drum.

A very interesting analogue is obtained if we remove the springs k from the differential as shown in Fig. 2. We then have a mechanical system similar to the rear end of an automobile which system is analogous to an inductively coupled electrical network such as a transformer. It is well known that if we suddenly open the primary of a transformer carrying current we obtain a large induced voltage in the secondary. Our analogy suggests a corresponding action occurring in the rear end of an automobile with the sudden braking of one wheel ($\omega_1 = 0$) when the clutch is out. Here

the inertia of the drive shaft (M_3 in Fig. 2) will produce a sudden torque in the other wheel tending to swerve the car. If we keep the engine engaged by leaving the clutch in, the swerving torque is rapidly damped out because of the frictional damping resistance acting on the drive shaft (f_3 in Fig. 2). This action is analogous to the rapid damping out of the transient secondary current in a loaded transformer with high resistance windings.

The method used above for analysis of the electrical and mechanical systems of two degrees of freedom may readily be extended to electrical and mechanical systems of several degrees of freedom. In fact it may be extended to include the general linear electrical network and its mechanical analogue.

In the electrical network of n degrees of freedom (i.e. n meshes) with voltage E impressed in mesh No. 1 the differential equations may be written

$$\begin{aligned}
a_{11}i_1 + a_{12}i_2 + a_{13}i_3 + \cdots + a_{1n}i_n &= E,\\
a_{21}i_1 + a_{22}i_2 + a_{23}i_3 + \cdots + a_{2n}i_n &= 0,\\
a_{31}i_1 + a_{32}i_2 + a_{33}i_3 + \cdots + a_{3n}i_n &= 0,\\
\cdots\cdots\cdots

diagonal of the system of equations (3). When $i \neq k$ the operator a_{ik} involves the mutual parameters L_{ik}, R_{ik} and S_{ik} which are respectively the negative of the sum of all the inductances, resistances, and elastances along the branch common to meshes i and k. The term $a_{ik}i_k$ therefore represents the counter voltage developed in mesh i by current i_k when all of the meshes except mesh k are considered as open-circuited. In the electrical network of n meshes these differential-integral a-operators may readily be determined by inspection of the network.

The solution of equations (3) for networks of several degrees of freedom is not difficult, though it may be tedious, especially when the transient solution is desired. The solution will not be given here.

In an analogous mechanical system of n degrees of freedom involving belt-connected bevel gear differentials as shown in Fig. 2 with torque F applied to flywheel No. 1 the differential equations of motion may be written

$$\begin{aligned}
a_{11}\omega_1 + a_{12}\omega_2 + a_{13}\omega_3 \cdots + a_{1n}\omega_n &= F, \\
a_{21}\omega_1 + a_{22}\omega_2 + a_{23}\omega_3 \cdots + a_{2n}\omega_n &= 0, \\
a_{31}\omega_1 + a_{32}\omega_2 + a_{33}\omega_3 \cdots + a_{3n}\omega_n &= 0, \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot & \\
a_{n1}\omega_1 + a_{n2}\omega_2 + a_{n3}\omega_3 \cdots + a_{nn}\omega_n &= 0.
\end{aligned} \tag{4}$$

The left hand member of any one of these equations represents the sum of the counter torques acting upon the flywheel corresponding to the first number in the subscript of the a's in that equation. The right hand member is the applied torque acting upon that flywheel. The a-operators may be represented by the defining equation

$$a_{ik} = \left\{ K_{ik}\frac{d}{dt} + f_{ik} + k_{ik}\int dt \right\},$$

where the indices i and k may take on all integral values from 1 to n. When $i = k$ the operator a_{ii} or a_{kk} involves the parameters K_{ii}, f_{ii} and k_{ii} which are respectively the sum of all the flywheel moments of inertia, the damping constants, and the spring constants involved in the motion of flywheel M_i and its associated coupling flywheels (e.g. flywheel M_3 in Fig. 2) when all of the others are considered as clamped.

The term $a_{ii}\omega_i$ therefore represents the counter torque acting on flywheel M_i when all of the others with the exception of the coupling flywheels associated with M_i are considered as clamped. It will be observed that all of these terms lie along the principal diagonal of the system of equations (4). When $i \neq k$ the operator a_{ik} involves the mutual parameters K_{ik}, f_{ik}, and k_{ik} which are respectively the negative of the flywheel moment of inertia, the damping constant, and the spring constant associated with the flywheel which couples flywheels i and k (e.g. M_3 in Fig. 2). The term $a_{ik}\omega_k$ therefore represents the counter torque acting on flywheel i when all the flywheels are considered clamped except flywheel k and the one which couples flywheels k and i. In the mechanical system of n degrees of freedom involving combinations of bevel gear differentials as shown in Fig. 2 these differential-integral a-operators may readily be determined by inspection of the mechanical system. The analogy between the two systems is indicated in the identity of form of the equations (4) for the mechanical system with equations (3) for the general linear electrical network.

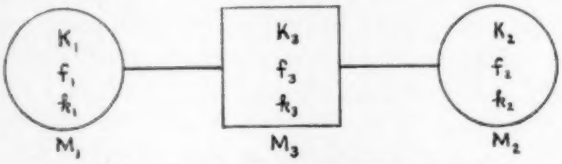
It should be remarked that the equations of motion (4) for a mechanical system of n degrees of freedom may apply to other mechanical arrangements which may involve linear vibrations and give a different interpretation to the coefficients. We are not concerned here with these other mechanical arrangements.

With reference to the mechanical systems described in this paper it should be noted that the applied torque is not necessarily harmonic and that it may be applied to any one of the several flywheels, or that different torques may simultaneously be applied to the several flywheels.

We shall now show how mechanical systems analogous to given electrical networks may be set up. Two analogues will be given and diagrams shown illustrating others. For this purpose it will be convenient to use a simple symbol to designate the bevel gear differential with or without its associated flywheels, damping vanes, or springs. This symbol will be as shown in Fig. 3, where the circles correspond to the flywheels M_1 and M_2 and the square corresponds to the coupling flywheel M_3 of Fig. 2. In the circles or square will

be placed the letters K, f, or k depending upon whether the moving elements include a heavy flywheel, a damping vane, or a spring respectively. When the circles or square include

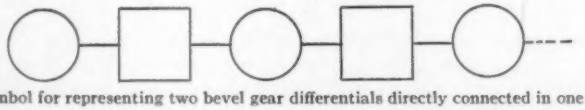
FIG. 3.



A symbol for representing the bevel gear differential of Fig. 2.

no letters they will represent light, relatively inertia-free drums. This symbol with appropriate letters will therefore represent the analogue to any two mesh electrical network, or to a single mesh network if one of the flywheels is indicated as clamped. With combinations of these symbols we may represent analogues to electrical networks of more than two meshes. Lines joining any two circles will represent light flexible chain belts. When two (or more) of the differentials are directly connected in one line they will be drawn as in Fig. 4. In this case the positive direction of rotation of the

FIG. 4.



A symbol for representing two bevel gear differentials directly connected in one line.

flywheels of the even numbered (from left to right) differential gears directly connected in line will be opposite in direction from that given for the flywheels of the differential of Fig. 2.

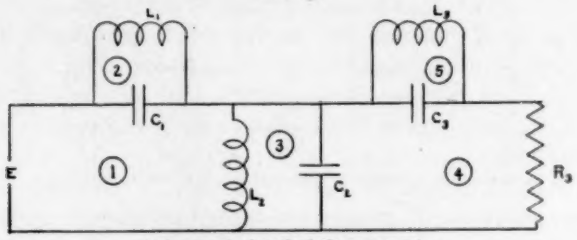
We shall illustrate the use of these symbols by setting up the mechanical system analogous to the electrical filter network shown in Fig. 5. In this network we number the meshes and tabulate the following:

Mesh No. 1 is coupled capacitatively to mesh No. 2 and inductively to mesh No. 3.
Mesh No. 3 is coupled capacitatively to mesh No. 4.
Mesh No. 4 is coupled capacitatively to mesh No. 5.

There are four common or mutual branches between meshes and we shall therefore need four bevel gear differentials for the mechanical analogue to this electrical network. We may

now draw the mechanical analogue lettering the flywheels or light drums which are symbolized as circles (not the coupling flywheels) as M_1 to M_5 corresponding respectively to the

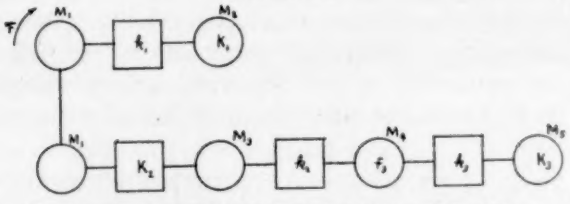
FIG. 5.



A five mesh electrical filter network.

meshes 1 to 5 of the electrical network. The mechanical analogue is shown in Fig. 6. For this figure we may tabulate

FIG. 6.



A mechanical analogue for the electrical filter network of Fig. 5.

the following corresponding to the tabulation for the electrical network:

M_1 is spring coupled to M_2 and inertia coupled to M_3,
M_3 is spring coupled to M_4,
M_4 is spring coupled to M_5.

The circles inclosing no letters and representing light, relatively inertia-free drums mounted in place of flywheels are used because all of the inductance of the analogous electrical mesh is in a common (mutual) branch. When the letter K is not inclosed in the squares they are understood to represent light, relatively inertia-free drums to which springs or damping vanes (indicated by k or f) are attached for spring or friction coupling respectively. When two (or more) circles representing flywheels or light drums are connected with lines indicating belt connections it is understood that the sum of their moments or inertia, damping constants, or spring constants corresponds to the sum of the inductances, resistances,

or elastances *exclusive* of the mutual branches in the analogous electrical mesh. These circles will include no letters (see M_1, Fig. 6) if all of the elements of the analogous electrical mesh (see mesh No. 1, Fig. 5) are included in mutual branches. The circle M_3 of Fig. 6 represents a relatively inertia-free drum since the corresponding mesh No. 3 of the electrical network of Fig. 5 has all of its elements in mutual branches. However, since mesh No. 4 in Fig. 5 includes the resistance R_3 which is not in a mutual branch, we have the corresponding light drum M_4 of Fig. 6 fitted with a damping vane f_3. The circles indicating flywheels or light drums will therefore inclose letters representing the mechanical elements analogous to the electrical elements on the contour of the corresponding mesh *exclusive of the common branches*. The squares representing coupling flywheels or light drums will always inclose letters representing the mode of coupling corresponding to the coupling elements of the analogous common branch of the electrical network. Referring to the mechanical system of Fig. 6 the torque F applied to the drums M_1 corresponds to the voltage E applied to mesh No. 1 of the electrical network of Fig. 5 and the angular velocity of the drum M_4 corresponds to the current through R_3 of this network.

The differential equations of motion for the mechanical system of Fig. 6 may be set up directly from the diagram. The system has five degrees of freedom and the differential-integral a-operators may be written down by inspection of the diagram:

$$a_{11} = \left\{ K_2 \frac{d}{dt} + k_1 \int dt \right\}, \qquad a_{12} = a_{21} = -k_1 \int dt,$$

$$a_{22} = \left\{ K_1 \frac{d}{dt} + k_1 \int dt \right\}, \qquad a_{13} = a_{31} = -K_2 \frac{d}{dt},$$

$$a_{33} = \left\{ K_2 \frac{d}{dt} + k_2 \int dt \right\}, \qquad a_{34} = a_{43} = -k_2 \int dt,$$

$$a_{44} = \{ f_3 + (k_2 + k_3) \int dt \}, \qquad a_{45} = a_{54} = -k_3 \int dt.$$

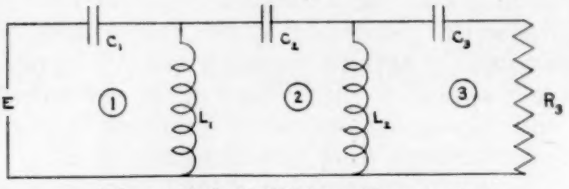
$$a_{55} = \left\{ K_3 \frac{d}{dt} + k_3 \int dt \right\},$$

The other a-operators are each equal to zero. The defining equation for the a-operators is

$$a_{ik} = \left\{ K_{ik} \frac{d}{dt} + f_{ik} + k_{ik} \int dt \right\},$$

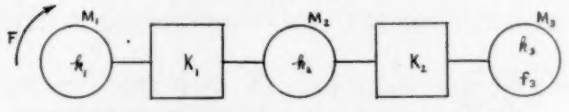
where i and k take on the values 1 to 5 for this mechanical system. When $i = k$ K_{ii}, f_{ii}, and k_{ii} represent respectively the sums of the moments of inertia, the damping constants, and the spring constants associated with the flywheels or drums M_i together with the moments of inertia, damping constants, and the spring constants of the coupling flywheels or drums on both sides of M_i. When $i \neq k$ K_{ik}, f_{ik}, and k_{ik} represent respectively the moment of inertia, the damping constant, and the spring constant of the flywheel or drum (with associated damping vane or spring) which couples M_i to M_k. The terms $a_{ik} = a_{ki}$ are negative as explained earlier in this paper. Substitution of these a-operators into equations (4) gives the differential equations of motion for the mechanical system of Fig. 6 analogous to the five mesh electrical filter network of Fig. 5.

FIG. 7.



A three mesh electrical high-pass filter network.

FIG. 8.



A mechanical analogue for the electrical filter network of Fig. 7.

The mechanical analogues for ladder or recurrent networks are particularly simple since the bevel gear differentials may all be arranged colinearly eliminating the need for belt connections. Fig. 7 shows a high-pass filter network of three meshes and Fig. 8 shows the mechanical analogue. The differential equations of motion for this mechanical high-pass filter are

$$a_{11}\omega_1 + a_{12}\omega_2 + a_{13}\omega_3 = F,$$
$$a_{21}\omega_1 + a_{22}\omega_2 + a_{23}\omega_3 = 0,$$
$$a_{31}\omega_1 + a_{32}\omega_2 + a_{33}\omega_3 = 0,$$

where the a-operators are

$$a_{11} = \left\{ K_1 \frac{d}{dt} + k_1 \int dt \right\},$$

$$a_{22} = \left\{ (K_1 + K_2) \frac{d}{dt} + k_2 \int dt \right\},$$

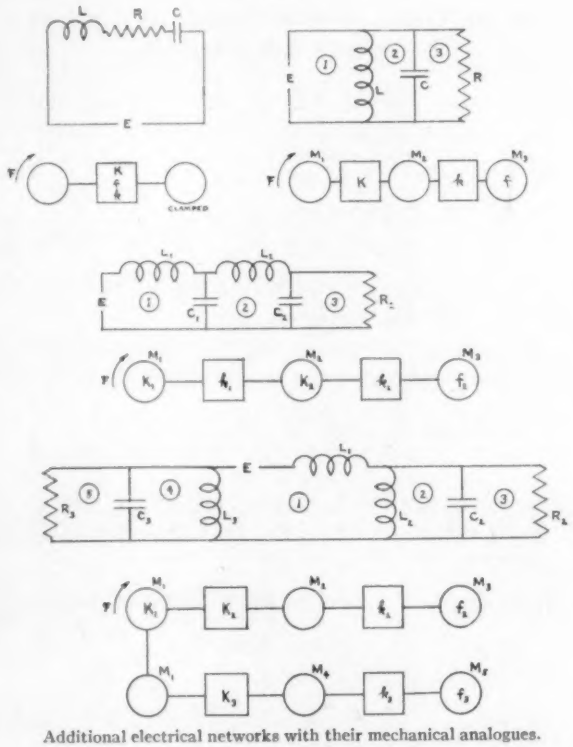
$$a_{33} = \left\{ K_2 \frac{d}{dt} + f_3 + k_3 \int dt \right\},$$

$$a_{12} = a_{21} = - K_1 \frac{d}{dt},$$

$$a_{13} = a_{31} = 0,$$

$$a_{23} = a_{32} = - K_2 \frac{d}{dt}.$$

FIG. 9.



Additional electrical networks with their mechanical analogues.

F is the torque applied to drum M_1 and ω_1, ω_2 and ω_3 are the angular velocities of drums M_1, M_2 and M_3 respectively. Fig. 9 shows a number of electrical networks together with their mechanical analogues.

The analysis of the electrical networks and their mechanical analogues given in this paper should enable the electrical engineer to construct a mechanical analogue to any linear electrical network or, conversely, to draw the electrical analogue for certain mechanical vibration systems. It is believed that this analysis will be of assistance to the electrical engineer who finds it necessary to extend his work into the field of mechanical vibrations.

APPENDIX.

DERIVATION OF THE EQUATIONS OF MOTION FOR THE BEVEL GEAR DIFFERENTIAL BY USE OF LAGRANGE'S EQUATIONS.

When M_1 and M_2 of Fig. 2 are moving with angular velocities ω_1 and ω_2 respectively the total kinetic energy of the system is

$$\begin{aligned} T &= \tfrac{1}{2}K_1\omega_1^2 + \tfrac{1}{2}K_2\omega_2^2 + \tfrac{1}{2}K_3(\omega_2 - \omega_1)^2 \\ &= \tfrac{1}{2}(K_1 + K_3)\omega_1^2 + \tfrac{1}{2}(K_2 + K_3)\omega_2^2 - K_3\omega_1\omega_2, \end{aligned} \tag{5}$$

where K_1, K_2 and K_3 are the moments of inertia of the flywheels M_1, M_2 and M_3 respectively.

When M_1 and M_2 are given angular displacements θ_1 and θ_2 respectively the total potential energy stored in the springs k is

$$\begin{aligned} V &= \tfrac{1}{2}k_1\theta_1^2 + \tfrac{1}{2}k_2\theta_2^2 + \tfrac{1}{2}k_3(\theta_2 - \theta_1)^2 \\ &= \tfrac{1}{2}(k_1 + k_3)\theta_1^2 + \tfrac{1}{2}(k_2 + k_3)\theta_2^2 - k_3\theta_1\theta_2, \end{aligned} \tag{6}$$

where k_1, k_2 and k_3 are the spring constants of the springs k.

It will be convenient to make the following substitutions:[5]

$$K_{11} = K_1 + K_3, \qquad k_{11} = k_1 + k_3, \qquad \dot{\theta} = \frac{d\theta}{dt} = \omega.$$

$$K_{22} = K_2 + K_3, \qquad k_{22} = k_2 + k_3,$$

$$K_{12} = K_{21} = -K_3, \qquad k_{12} = k_{21} = -k_3,$$

[5] Lord Rayleigh, loc. cit., p. 103.

With these substitutions equations (5) and (6) become

$$T = \tfrac{1}{2}K_{11}\dot{\theta}_1^2 + \tfrac{1}{2}K_{22}\dot{\theta}_2^2 + K_{12}\dot{\theta}_1\dot{\theta}_2, \tag{7}$$

$$V = \tfrac{1}{2}k_{11}\theta_1^2 + \tfrac{1}{2}k_{22}\theta_2^2 + k_{12}\theta_1\theta_2. \tag{8}$$

The total torque applied to the flywheel M_1 consists of the applied harmonic torque F plus the counter damping torque $-f_1\dot{\theta}_1$ proportional to the angular velocity of M_1 plus the damping torque $f_3(\dot{\theta}_2 - \dot{\theta}_1)$ proportional to the angular velocity of M_3.

The total torque applied to the flywheel M_2 consists of the counter damping torque $-f_2\dot{\theta}_2$ proportional to the angular velocity of M_2 plus the damping torque $-f_3(\dot{\theta}_2 - \dot{\theta}_1)$ proportional to the angular velocity of M_3.

We will represent the total torques acting upon M_1 and M_2 by Θ_1 and Θ_2 respectively. These may now be written as follows:

$$\Theta_1 = F - f_{11}\dot{\theta}_1 - f_{12}\dot{\theta}_2,$$
$$\Theta_2 = -f_{22}\dot{\theta}_2 - f_{21}\dot{\theta}_1,$$

where

$$f_{11} = f_1 + f_3,$$
$$f_{22} = f_2 + f_3,$$
$$f_{12} = f_{21} = -f_3.$$

Substituting these values of T, V and Θ into Lagrange's Equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_i}\right) - \frac{\partial T}{\partial \theta_i} + \frac{\partial V}{\partial \theta_i} = \Theta_i, \qquad i = 1, 2,$$

we obtain after simplifying

$$K_{11}\ddot{\theta}_1 + K_{12}\ddot{\theta}_2 + k_{11}\theta_1 + k_{12}\theta_2 = F - f_{11}\dot{\theta}_1 - f_{12}\dot{\theta}_2,$$
$$K_{22}\ddot{\theta}_2 + K_{21}\ddot{\theta}_1 + k_{22}\theta_2 + k_{21}\theta_1 = -f_{22}\dot{\theta}_2 - f_{21}\dot{\theta}_1.$$

With rearrangement and the substitutions

$$\omega = \dot{\theta},$$
$$\int \omega dt = \theta,$$

these equations may be written

$$\begin{aligned} K_{11}\dot{\omega}_1 + f_{11}\omega_1 + k_{11}\int\omega_1 dt + K_{12}\dot{\omega}_2 \\ + f_{12}\omega_2 + k_{12}\int\omega_2 dt = F, \\ K_{22}\dot{\omega}_2 + f_{22}\omega_2 + k_{22}\int\omega_2 dt + K_{21}\dot{\omega}_1 \\ + f_{21}\omega_1 + k_{21}\int\omega_1 dt = 0, \end{aligned} \tag{9}$$

or more concisely

$$\begin{aligned} a_{11}\omega_1 + a_{12}\omega_2 &= F, \\ a_{21}\omega_1 + a_{22}\omega_2 &= 0, \end{aligned} \tag{10}$$

where the a's are the differential-integral operators [6]

$$a_{11} = \left\{ K_{11}\frac{d}{dt} + f_{11} + k_{11}\int dt \right\},$$

$$a_{12} = a_{12} = \left\{ K_{12}\frac{d}{dt} + f_{12} + k_{12}\int dt \right\},$$

$$a_{22} = \left\{ K_{22}\frac{d}{dt} + f_{22} + k_{22}\int dt \right\}.$$

Equations (10) are the differential equations of motion for the bevel gear differential with associated springs, damping vanes, and flywheels as shown in Fig. 2.

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[6] Lord Rayleigh, loc. cit., p. 104.

TENSORS ASSOCIATED WITH A PAIR OF QUADRATIC DIFFERENTIAL FORMS.

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ABSTRACT.

A pair of quadratic differential forms in n dimensions determine at least partly at each point of space, a system of n invariants and a set of n orthogonal unit vectors, with a one to one correspondence between the two sets. It is found very convenient to use these invariants and vectors as auxiliary quantities in terms of which tensors associated with the two forms can be expressed. In order that a tensor, which is a rational function [1] of these auxiliary quantities and their higher derivatives, be also a rational function of the components of the original two forms and their higher derivatives, it is necessary and sufficient that this tensor be invariant under a certain group of transformations on the auxiliary quantities. The Galois theory of groups is applied and a generalization to tensors is obtained of the theorem that every rational symmetric function of the roots of an algebraic equation is a rational function of the coefficients of the equation. The tensors that are rational functions of the components of the original two forms are considered in detail. Certain systems of fundamental tensors, symmetric relative to the auxiliary quantities, are introduced, such that every rational tensor function of the auxiliary quantities and symmetric relative to these quantities, can be expressed uniquely in terms of these fundamental tensors. Only tensors of even order are rational functions of the two forms. If one form is invariantively related to the other, the theory here developed becomes a theory for a single form.

1. INTRODUCTION.

Let $g_{\alpha\beta}dx^\alpha dx^\beta$ and $G_{\alpha\beta}dx^\alpha dx^\beta$ where $(\alpha, \beta = 1 \cdots n)$ represent two real quadratic differential forms. The theory here developed is a purely algebraic theory which holds at any particular point of the space, where the assumptions made, are valid. When derivatives are spoken of, their existence is only assumed. We shall assume in this paper that the two forms are non singular and that the elementary divisors of the

[1] A tensor that is a rational function of a set of quantities is defined as a tensor whose components are rational functions of this set of quantities, the functional relationship being invariant under coördinate transformations.

pencil $G_{\alpha\beta} - \lambda g_{\alpha\beta}$ are simple at the point considered. Then

$$g_{\alpha\beta} = \sum_{m}^{1-n} e_m g_{m\alpha} g_{m\beta},$$
$$G_{\alpha\beta} = \sum_{m}^{1-n} e_m \lambda_m g_{m\alpha} g_{m\beta}. \tag{1, 1}$$

The $\lambda_m (m = 1 \cdots n)$ are the n roots of the equation:

$$\text{det.}\ (G_{\alpha\beta} - \lambda g_{\alpha\beta}) = 0. \tag{1, 2}$$

They are n functions of position, invariant under coördinate transformations. A one to one correspondence can be set up between the $\lambda_m(m)$ and a set of unit vectors $(g_1 \cdots g_n)$ where $g_m = (g_{m\alpha}, \alpha = 1 \cdots n)$ (m). The $g_{m\alpha}$ satisfy the relations:

$$g^{\alpha\beta} g_{m\alpha} g_{n\beta} = e_m \delta_{mn}(m, n),$$

where the $e_m(m)$ are either $+1$ or -1.

Throughout this paper, Greek indices will be used to represent tensor components, and Latin indices to represent invariants and fundamental vectors and tensors associated with them. When permutations of the $\lambda_m(m)$ are considered, the one to one correspondence between the $\lambda_m(m)$, the $g_m(m)$ and the $e_m(m)$ will not be destroyed. Therefore when one speaks of a permutation of the $\lambda_m(m)$ it will be understood that the $g_m(m)$ and $e_m(m)$ are to be permuted in the same way.

The matrix $g_m{}^\alpha$ is defined by the relations:

$$g_m{}^\alpha g_{k\alpha} = e_m g_{m,\,k}(m, k). \tag{1, 4}$$

It should be noted that both g_{mk} and $g_\alpha{}^\beta$ represent the Kronecker δ, but have different interpretations. The first is a matrix of invariants, while the second is a mixed tensor. Equations corresponding to (1, 1) for the contravariant components are:

$$g^{\alpha\beta} = \sum_{m} e_m g_m{}^\alpha g_m{}^\beta,$$
$$G^{\alpha\beta} = \sum_{m} e_m \lambda_m g_m{}^\alpha g_m{}^\beta. \tag{1, 5}$$

In no case are the $\lambda_m(m)$ and $g_{m\alpha}(m, \alpha)$ uniquely determined. When one set is given, however, all the other possibilities can be obtained by throwing the original set under a

group of transformations. Or stated in another way, the relations (1, 1) are invariant under a certain group of transformations G on the $\lambda_m(m)$ and $g_{m\alpha}(m, \alpha)$. We shall restrict ourselves to those functions of the $\lambda_m(m)$ and $g_{m\alpha}(m, \alpha)$ which are tensors and at the same time are single valued functions of the $g_{\alpha\beta}, G_{\alpha\beta}$ and their higher derivatives. This requires that the tensors built out of the $\lambda_m(m)$ and $g_{m\alpha}(m, \alpha)$ be also invariant under the group G. In this connection we shall make use of the following theorem in the Galois theory of Groups:

Theorem G: Let $K(U_1 \cdots U_r)$ be a field generated by the elements $(U_1 \cdots U_r)$. Consider an extension of this field by the addition of elements $(V_1 \cdots V_s)$. Assume that the $(U_1 \cdots U_r, V_1 \cdots V_s)$ satisfy a set of rational algebraic relations $(R_1 \cdots R_p)$. Let G_1 be the Galois group of transformations of the $(V_1 \cdots V_s)$ which leave the $(R_1 \cdots R_p)$ invariant. If a rational function $R(U_1 \cdots U_r, V_1 \cdots V_s)$ is invariant under G_1, then it can be expressed as a rational function of the $(U_1 \cdots U_s)$.

The following theorem can now be proved:

Theorem I. The class of all rational functions of the $g_{\alpha\beta}, G_{\alpha\beta}(\alpha, \beta)$ and their higher derivatives up to the rth order is identical with the class of all rational functions of the λ_m, $\sqrt{e_m} g_{m\alpha}(m, \alpha)$ and their higher derivatives up to the rth order which are invariant under the group G. Likewise the class of all rational tensor functions of the $g_{\alpha\beta}, G_{\alpha\beta}(\alpha, \beta)$ and their higher derivatives up to those of the rth order is identical with the class of all rational tensor functions of λ_m, $\sqrt{e_m} g_{m\alpha}(m, \alpha)$ and their derivatives up to the rth order which are invariant under the group $(G.)$

Proof: We shall make use of theorem G. The field K is in this case the field generated by the functions $g_{\alpha\beta}, G_{\alpha\beta}(\alpha, \beta)$ and their higher derivatives up to those of the rth order. The λ_m, $\sqrt{e_m} g_{m\alpha}(m, \alpha)$ and their higher derivatives up to the rth order correspond to the $(V_1 \cdots V_s)$. The $(R_1 \cdots R_p)$ correspond to the relations (1, 1) and the further relations obtained by differentiating these successively. The group G in the theorem I corresponds to the group G_1. Theorem I follows at once.

2. THEOREM I AND THE FUNDAMENTAL FORMULÆ WHEN THE $\lambda_m(m)$ ARE ALL UNEQUAL.

In this case the group G is the group of all permutations of the $\lambda_m(m)$ extended by the group reflections $g_m \rightarrow \pm g_m'$. Theorem I becomes in this case:

Theorem (2, 1). A tensor which is a rational function of the λ_m, $\sqrt{e_m} g_{m\alpha}(m, \alpha)$ and their higher derivatives up to the rth order, and which is invariant under the group of permutations of the $\lambda_m(m)$ extended by the group of reflections $g_m \rightarrow \pm g_m'(m)$, is also a rational function of the $g_{\alpha\beta}$, $G_{\alpha\beta}(\alpha\beta)$ and their higher derivatives up to the rth order.

This theorem is a generalization of a well known theorem in the theory of the symmetric functions of the roots of an algebraic equation.

Certain fundamental formulæ are needed for the transformation $(\lambda_m, g_{m\alpha}(m, \alpha))$ to $(g_{\alpha\beta}, G_{\alpha\beta}(\alpha, \beta))$. Define $\overset{r}{G}_{\alpha\beta}(r = 0 \cdots n-1)$ by the recursion formulæ:

$$\overset{r}{G}_{\alpha\beta} = \overset{r-1}{G}_{\alpha}{}^{\gamma} G_{\gamma\beta}(r, \alpha\beta), \quad \overset{0}{G}_{\alpha\beta} = g_{\alpha\beta}\ (\alpha, \beta). \qquad (2, 1)$$

Define $\lambda_{r,m} = \lambda_m{}^r$. The matrix $\lambda_{r,m}\ (r, m)$ is then the well known alternant:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \cdots \\ \lambda_1{}^2 & \lambda_2{}^2 & \lambda_3{}^2 & \lambda_4{}^2 & \cdots \\ \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \end{bmatrix}.$$

Its reciprocal $\mu_{m,s}(m, s)$ is defined by the equation:

$$\sum_{m}^{1-n} \lambda_{r,m} \mu_{m,s} = g_{rs}. \qquad (r, s)\ (2, 2)$$

Then

$$\overset{r}{G}_{\alpha\beta} = \sum_{m}^{1-n} \epsilon_m \lambda_m{}^r g_{m\alpha} g_{m\beta}. \qquad (\alpha, \beta)\ (2, 3)$$

Solving, one obtains:

$$g_{m\alpha} g_{m\beta} = \epsilon_m \sum_{r}^{1-n} \mu_{m,r} \overset{r}{G}_{\alpha\beta} \qquad (m, \alpha, \beta)\ (2, 4)$$

and therefore

$$g_{m\alpha} = \eta_m \sqrt{e_m \sum_r^r \mu_{m,r} G_{\alpha\alpha}} \qquad (m, \alpha), (2, 5)$$

where $\eta_m = \pm 1(m)$ is an arbitrary function of m.

Likewise one obtains corresponding equations for the contravariant components:

$$g_m{}^{\alpha} = \eta_m \sqrt{e_m \sum_r^{1-n} \mu_{m,r} G^{\alpha\alpha}}. \qquad (2, 6)$$

Equations for the transformations of the derivatives can be easily obtained. For instance in the case of the first derivatives, one has the following equations to solve:

$$\begin{gathered} \sum_m^{1-n} e_m \left(g_{m\alpha} \frac{\partial g_{m\beta}}{\partial x^\gamma} + \frac{\partial g_{m\alpha}}{\partial x^\gamma} g_{m\beta} \right) = \frac{\partial g_{\alpha\beta}}{\partial x^\gamma} (\alpha, \beta, \gamma), \\ \sum_m^{1-n} e_m \frac{\partial \lambda_m}{\partial xy} g_{m\alpha} g_{m\beta} + \sum_m^{1-n} e_m \lambda_m \left(g_{m\alpha} \frac{\partial g_{m\beta}}{\partial x^\gamma} + \frac{\partial g_{m\alpha}}{\partial x^\gamma} g_{m\beta} \right) \\ = \frac{\partial G_{\alpha\beta}}{\partial x^\gamma} (\alpha, \beta, \gamma). \end{gathered} \qquad (2,7)$$

The solutions of (2, 7) are

$$\begin{gathered} \frac{\partial \lambda_m}{\partial x^\gamma} = e_m g_m{}^{\epsilon} \frac{\partial G_{\epsilon\eta}}{\partial x^\gamma} g_m{}^{\eta} - e_m \lambda_m g_m{}^{\epsilon} \frac{\partial g_{\epsilon\eta}}{\partial x^\gamma} g_m{}^{\gamma}, \\ \frac{\partial g_{m\alpha}}{\partial x^\gamma} = \sum_l^{\neq m} \frac{e_m}{\lambda_m - \lambda_l} \left\{ g_l{}^{\epsilon} \frac{\partial G_{\epsilon\eta}}{\partial x^\gamma} g_m{}^{\gamma} - \lambda_l g_l{}^{\epsilon} \frac{\partial g_{\epsilon\eta}}{\partial x^\gamma} g_m{}^{\eta} \right\} g_l{}^{\alpha} \\ + \tfrac{1}{2} e_m g_{m\alpha} g_m{}^{\epsilon} \frac{\partial g_{\epsilon\eta}}{\partial x^\gamma} g_m{}^{\eta} (m, \alpha, \gamma). \end{gathered} \qquad (2, 8)$$

In the same way the pth derivatives of the λ_m, $g_{m\alpha}(m, \alpha)$ can be expressed as functions of the pth derivatives of $g_{\alpha\beta}$, $G_{\alpha\beta}(\alpha, \beta)$ and the derivatives up to the $p-1$th order of λ_m, $g_{m\alpha}(m, \alpha)$. Then by successive reductions, the pth derivatives of the λ_m, $g_{m\alpha}(m, \alpha)$ can be expressed as rational functions of the $\lambda_m g_{m\alpha}(m, \alpha)$ and the derivatives up to the pth order of

$(g_{\alpha\beta}, G_{\alpha\beta})$ (α, β). Thus:

$$\frac{\partial^r \lambda_m}{\partial^{r_1} x^{(1)} \cdots \partial^{r_s} x^{(s)}} = \Lambda_{m, r_1 \cdots r_s}\left(\lambda_l, g_{la}, \frac{\partial^s g_{\alpha\beta}}{\partial^{s_1} x^{(1)} \cdots \partial^{s_t} x^{(t)}}, \frac{\partial^r G_{\alpha\beta}}{\partial^{s_1} x^1 \cdots \partial s^t x^t}, l, \alpha, s_1 \cdots s_t, \alpha\beta\right),$$

$$\frac{\partial^r g_{m\alpha}}{\partial^{r_1} x^{(1)} \cdots \partial^{r_s} x^s} = G_{m, \alpha, r_1 \cdots r_s}\left(\lambda_l, g_{la}, \frac{\partial^s g_{\alpha\beta}}{\partial^{s_1} x^{(1)} \cdots \partial^{s_t} x^t}, \frac{\partial^s G_{\alpha\beta}}{\partial^{s_1} x^1 \cdots \partial^{s_t} x^t}, l, \alpha, s_1 \cdots s_t, \alpha, \beta\right). \quad (2, 9)$$

The following theorem can now be proved:

Theorem (2, 1). Given a tensor which is a rational function of the λ_m, $\sqrt{e_m} g_{m\alpha}(m, \alpha)$ and their higher derivatives, and which is invariant under the group G. The computation of the components of this tensor as a function of the $g_{\alpha\beta}$, $G_{\alpha\beta}(\alpha, \beta)$ and their higher derivatives is reduced by equations (2, 4), (2, 8) and (2, 9) to the computation of certain symmetric functions of the $\lambda_m(m)$.

Proof: By equations (2, 9) any component C of this tensor can be reduced to the form $C = \frac{P}{Q}$ where P and Q are each of the form $\sum_k F_k(\lambda_m, g_{m\alpha}) H_k$. The H_k are integral rational functions of the $g_{\alpha\beta}$, $G_{\alpha\beta}(\alpha\beta)$ and their higher derivatives, and are therefore invariant under the group G. The F are integral rational functions of λ_m, $\sqrt{e_m} g_{m\alpha}(m, \alpha)$ but not necessarily invariant under G. However, $C = \frac{1}{n!} \sum \frac{P}{Q}$ where the summation is over all permutations of the $\lambda_m(m)$. This sum can again be reduced again to the original form $C = \frac{P'}{Q'}$, where the new F_k' are rational integral functions invariant under G. They are therefore by theorem (1, 2) rational functions of $g_{\alpha\beta}$, $G_{\alpha\beta}(\alpha, \beta)$, and then by equations (1, 1) are expressible as rational functions of $(\lambda_m, e_m g_{m\alpha} g_{m\beta})(m, \alpha\beta)$. In this form, equations (2, 4) will reduce them to functions of $\lambda_m(m)$, $G_{\alpha\beta}(r, \alpha\beta)$, invariant under the group G. The problem is therefore re-

duced to the computation of certain symmetric functions of $\lambda_m(m)$.

3. THEOREM I WHEN THE $\lambda_m(m)$ ARE NOT ALL UNEQUAL.

We shall modify the notation somewhat, and represent a multiple root by a single symbol λ_a where $(a = 1 \cdots n_0)$. Let f_a be the multiplicity of the root λ_a, η_f the number of roots of multiplicity f. Then

$$\eta = \sum_{a}^{1-n_0} f_a = \sum_{f}^{1-fn_0} f\eta_f. \tag{3, 1}$$

It will be convenient to arrange the order of the $\lambda_a(a)$ so that f_a is an increasing monatonic function of a. Then the $\lambda_a(a)$ of the same multiplicity appear together in groups. To each λ_a corresponds a space S_a of f_a dimensions, in which a system of f unit orthogonal vectors can be constructed:

$$g_{a_i} = (g_{a_i\alpha}, \alpha = 1 \cdots n)(i = 1 \cdots f_a).$$

Define $\mu_{b,r}$, $g_{a_i}{}^{\alpha}$, $g_{a,\alpha\beta}$ and $\lambda_{r,a}$ as follows:

$$\begin{aligned} \lambda_{r,a} &= \lambda_a{}^r && (a, r) \\ \sum_{r}^{1-n_0} \lambda_{a,r}\mu_{r,b} &= g_{a,b}, && (a, b) \quad (3, 2) \\ g_{a_i}{}^{\alpha} g_{b_j\alpha} &= e_a g_{a_i b_j}. && (a_i, b_j) \end{aligned}$$

The following relations can then easily be proved:

$$g_{a,\alpha\beta} g_b{}^{\beta\gamma} = e_a g_{ab} g_{a,}{}^{\gamma} \qquad (ab, \alpha, \gamma), \ (3, 3)$$

$$g_{a,\alpha\beta} g_b{}^{\alpha\beta} = e_a g_{ab} f_{ab} \qquad (a, b), \ (3, 4)$$

$$\overset{(r)}{G}_{\alpha\beta} = \sum_{a}^{1-n_0} e_a \lambda_a{}^r g_{a,\alpha\beta} (r = 1 \cdots n_0) \ (\alpha, \beta), \quad (3, 5)$$

$$\overset{(r)}{G} = \overset{r}{G}_{\alpha\beta} g^{\alpha\beta} = \sum_{a}^{1-n_0} \lambda_a{}^r f_a = s_r\,(r),^2 \quad (3, 6)$$

$$g_{a,\alpha\beta} = e_a \sum_{r}^{1-n_0} \mu_{a,r} \overset{r}{G}_{\alpha\beta}. \qquad (a, \alpha\beta) \ (3, 7)$$

The group G of theorem I is in this case the direct product certain groups P_f $(f = 1 \cdots f_{n0})$ and D_a $(a = 1 \cdots n_0)$, where

[2] s_r is taken from the notation of elementary symmetric function theory.

P_f is the group of all permutations of the λ_a of multiplicity f_a, and D_a the group of all orthogonal transformations in S_a.

4. TENSORS THAT ARE RATIONAL ALGEBRAIC FUNCTIONS OF THE $g_{\alpha\beta}$ AND $G_{\alpha\beta}$.

It will not be assumed that the $\lambda_m(m)$ are all unequal. The following lemmas are well known: Lemma (4, 1): Any analytic function $F(\lambda_a, g_{ma}(m, \alpha))$ invariant under coördinate transformations is a function of the $\lambda_a(a)$ only. Lemma (4, 2): Any tensor of the type $K_{\alpha\beta\ldots}{}^{\epsilon\eta\cdots}$ is expressible uniquely in terms of a fundamental system of vectors $g_{a_i a}(a_i\alpha)$ thus:

$$K_{\alpha\beta\ldots}{}^{\epsilon\eta\cdots} = \sum_{a_i\cdots d_k} K_{a_i\cdots d_k} g_{a_i\alpha}\cdots g_{d_k}{}^{\epsilon}\cdots,$$

where the $Ka_i\ldots_{d_k}$ are invariants. If this tensor is a rational algebraic function of $g_{\alpha\beta}$, $G_{\alpha\beta}(\alpha, \beta)$, these invariants are by lemma (4, 1) functions of the $\lambda_a(a)$ only.

The following theorems follow immediately:

Theorem (4, 3). A tensor of odd order is not a rational algebraic function of $g_{\alpha\beta}$, $G_{\alpha\beta}$.

Proof: An odd order tensor is expressible in the form given in lemma (4, 2). By theorem I this tensor must be invariant under the group of reflections $g_m \rightarrow g_m'(m)$, which is a sub group of G. As the fundamental vectors appear multiplied together in sets of odd number, these products will not remain invariant in sign under these reflections. If the tensor is to remain invariant, the invariants $K_{\alpha_i\ldots d_k}$ of the lemma must change sign under certain reflections. But this is impossible as these invariants are functions only of the $\lambda_a(a)$. This completes the proof.

Theorem (4, 4). Any tensor that is a rational function of the $g_{\alpha\beta}$ and $G_{\alpha\beta}(\alpha, \beta)$ can be constructed rationally out of the elements λ_a, $e_a g_{a,\alpha\beta}$. Furthermore any tensor that is a rational function of these elements and that is invariant under the groups $P_f(f)$ is a rational tensor function of the $g_{\alpha\beta}$ and $G_{\alpha\beta}(\alpha, \beta)$.

Proof: From equations (3, 6) it follows that $g_{\alpha\beta}$ and $G_{\alpha\beta}$ are rational algebraic functions of $\lambda_a(a)$, $e_a g_{a,\alpha\beta}$. The first statement in the theorem follows immediately. The second statement follows as a consequence of theorem I and the fact

that the elements $e_a g_{a,\alpha\beta}(a, \alpha, \beta)$ themselves are invariant under the groups $S_a(a)$.

Theorem (4, 5). Any rational algebraic invariant of $g_{\alpha\beta}$ and $G_{\alpha\beta}$ is expressible as a function of the $\lambda_a(a)$ which is invariant, under the groups $P_f(f)$. Conversely, any function of the $\lambda_a(a)$ which is invariant under the $P_f(f)$ is a rational algebraic invariant of $g_{\alpha\beta}$ and $G_{\alpha\beta}$.

This theorem follows at once from lemmas (4, 1) (4, 2) and theorem (4, 1).

Theorem (4, 6). A tensor that is a rational algebraic function of $g_{\alpha\beta}$ and $G_{\alpha\beta}(\alpha, \beta)$ is of even order and expressible as the sum of tensors of the type

$$K_{\alpha\beta\ldots\epsilon\eta} = \sum_{a-d} e_a \cdots e_d K_{a\ldots d} g_{a,\alpha\beta} \cdots g_{d,\epsilon\eta},$$

where the $K_{a\ldots d}$ are rational functions of the $(a, d = 1 \cdots n_0)$. Conversely, any tensor so expressible which is invariant under the groups $P_f(f)$, is a rational algebraic function of the $g_{\alpha\beta}$ and $G_{\alpha\beta}(\alpha, \beta)$. This theorem is a consequence of lemma (4, 2), theorem (4, 4) and theorem (4, 5).

5. Fundamental symmetric tensors.

The theory of symmetric functions of the roots of an algebraic equation can easily be extended to the case where the roots of the equation are not simple. In this case, a function of the $\lambda_a(a)$ will be called symmetric, if it is invariant under the transformations of the groups $P_f(f)$. It is very easy to show that the n_f roots λ_a of multiplicity f_a are the roots of an algebraic equation E_f of degree n_f, whose coefficients lie in the field of the rational functions generated by $g_{\alpha\beta}$ and $G_{\alpha\beta}(\alpha, \beta)$. Define $s_{f,r} = \sum_a^{fa=f} \lambda_a{}^r$. Then $s_{f,r}(r = 1 \cdots n_f)$ represent a complete system of fundamental symmetric functions for the equation E_f. We can now prove the theorem:

Theorem (5, 1). A rational function $F(\lambda_{(a)}, a = 1 \cdots n_0)$ which is invariant under the groups $P_f(f)$ can be expressed uniquely in terms of the fundamental symmetric functions $s_{f,r}$ $(f = 1 \cdots f_{n_0}, r = 1 \cdots n_f)$.

Proof: Treat the function F as a function only of the roots of multiplicity 1, the remaining $\lambda_a(a)$ being given certain

fixed values. It follows from the fundamental theorem in symmetric functions that F is a function of $s_{1,r}(r)$ and the $\lambda_a(a)$ multiplicity greater than 1. In the same way, this function can be treated as a function of the roots of multiplicity 2, the $s_{1,r}(r)$ and the $\lambda_a(a)$ of multiplicity greater than 2 being considered as constant. Then it follows that F is a function of $s_{1,r}$, $s_{2,r}$, $\cdots$ (r) and the roots of multiplicity greater than 2. In this way by successive reductions, it can be shown that F can be expressed as a function of the $(s_{1r}, s_{2r}, \cdots (r))$ only. Furthermore, assume two functions $F_1(s_{1,r}, s_{2r} \cdots (r))$ and $F_2(s_{1,r}, s_{2,r} \cdots (r))$, both of which are equal to the original $F(\lambda_a(a))$ for all (λ_a). Treat the F_1 and F_2 as functions of the roots of multiplicity f. From the theory of symmetric functions, it follows that $F_1 = F_2(s_{f,r}(r))$. This proves the uniqueness of the representation, and completes the proof of the theorem.

Theorem (5, 2). Every rational tensor function $K_{\alpha\beta}$ of the second order of $g_{\alpha\beta}$, $G_{\alpha\beta}$ (α, β) can be expressed uniquely in the form:

$$K_{\alpha\beta} = \sum_{h}^{1-n_0} S_h \overset{r}{G}_{\alpha\beta}$$

where S_h are functions of the $s_{f,r}(f, r)$. Conversely, every tensor of this form is a rational function of $g_{\alpha\beta}$ and $G_{\alpha\beta}(\alpha, \beta)$. The S_h are given by the formula:

$$S_h = \overset{h}{H}{}^{\alpha\beta} K_{\alpha\beta}$$

where $\overset{r}{H}_{\alpha\beta}(r)$ is a system of tensors adjoint to the system $\overset{r}{G}_{\alpha\beta}(r)$, and are given by the equations:

$$\overset{(r)}{H}_{\alpha\beta} = \sum_{a}^{1-n_0} \frac{e_a}{f_a} \mu_{ar} g_{a,\alpha\beta} \quad (r, \alpha\beta)$$

so that

$$\overset{r}{H}_{\alpha\beta} \overset{s}{G}{}^{\alpha\beta} = g_{r,s}(r, s).$$

Proof: Combining the equations (3, 7) with the representation given in theorem (4, 6), one obtains the representation

given above if

$$S_h = \sum_a^{1-n_0} e_a \mu_{ah} K_a.$$

The uniqueness of the representation follows as a direct consequence of the formula given above for the $S_{h\ldots k}$.

The $s_{f,r}(f, r)$ and $\overset{r}{G}_{\alpha\beta}(r, \alpha\beta)$ form a set of fundamental symmetric tensors of the oth and 2nd orders, in terms of which all rational tensor functions of the zeroth and second order of $g_{\alpha\beta}(\alpha\beta)$ and $G_{\alpha\beta}(\alpha\beta)$ can be uniquely expressed.

There are other sets of fundamental symmetric tensors which may be used for unique representations of the type of theorem (5, 2). If one considers the set $s_{f,r}(f, r)$ and $\overset{r}{G}_{\alpha\beta}(f, r)$ for the nondegenerate case where the $\lambda_m(m)$ are all different, and passes continuously to the case where they are not all different, this set splits up into four possible ones:

$$A: \; s_r \quad \overset{r}{G}_{\alpha\beta} \quad (r)$$

$$B: \; s_{f,r} \quad \overset{r}{G}_{\alpha\beta} \quad (f, r)$$

$$C: \; s_r \quad \overset{(fr)}{G}_{\alpha\beta} \quad ((fr), r)$$

$$D: \; s_{f,r} \quad \overset{(fr)}{G}_{\alpha\beta} \quad ((fr)),$$

where

$$s_r = \sum_a^{1\cdots n_0} \lambda_a{}^r f_a (r = 1 \ldots n),$$

$$s_{f,r} = \sum_a^{fa=f} \lambda_a{}^r (f = 1 \ldots f n_0,\ r = 1 \ldots n_f),$$

$$\overset{(rf)}{G}_{\alpha\beta} = \sum_a^{fa=f} C_a \lambda_a{}^r g_a,\ \alpha\beta \ ((rf), \alpha\beta).$$

The pair of indices (rf) is here treated as a single index with values $(1 \ldots n_0)$. The relationship between the $\overset{r}{G}_{\alpha\beta}(r = 1 \ldots n_0)$ and $\overset{(rf)}{G}_{\alpha\beta}((rf) = 1 \ldots n_0)$ is given by the fol-

lowing equations:

$$\overset{(rf)}{G}_{\alpha\beta} = \sum_{s}^{1\cdots n_0} \mu_{(fr),\,s} \overset{s}{G}_{\alpha\beta} \quad ((rf))(\alpha, \beta),$$

$$\overset{s}{G}_{\alpha\beta} = \sum_{(rf)}^{1\cdots n_0} V_{s,\,(fr)} \overset{(fr)}{G}_{\alpha\beta} \quad (S, \alpha\beta),$$

where

$$\mu_{(fr),\,s} = \sum_{a}^{fa=f} \lambda_{r,\,a} \mu_{a,\,s} \quad ((fr), s)$$

and its reciprocal $\nu_{t,\,(fr)}$ is defined by the equations

$$\sum_{(fr)}^{1-n_0} \nu_{t,\,(fr)} \mu_{(fr),\,s} = g_{t,\,s} \quad (t, s).$$

Since the s_{fr} are uniquely expressible in terms of the $\overset{(fr)}{G}_{\alpha\beta}$, it follows that every rational tensor function of $g_{\alpha\beta}$, $G_{\alpha\beta}(\alpha\beta)$ is uniquely expressible in terms of the $\overset{r}{G}_{\alpha\beta}(r)$ or of the $\overset{(rf)}{G}_{\alpha\beta}((rf))$.

It should be noted that n of the $s_f(f)$ are required, and only n_0 of the s_{fr}. On the other hand the former while being of higher degree in $\lambda_a(a)$, are more simply expressible in terms of the $g_{\alpha\beta}$, $G_{\alpha\beta}$. Likewise, the $\overset{(rf)}{G}_{\alpha\beta}$ are of lower degree in $\lambda_a(a)$ but are more complicated functions of the $g_{\alpha\beta}$,$G_{\alpha\beta}(\alpha\beta)$ than the $\overset{r}{G}_{\alpha\beta}(r)(\alpha\beta)$.

One can furthermore replace the $\overset{r}{G}_{\alpha\beta}$ and $\overset{(rf)}{G}_{\alpha\beta}(r, (rf))$ by orthogonalized systems. We shall illustrate this by orthogonalizing the system $\overset{r}{G}_{\alpha\beta}(r)$. Represent the orthogonalized system by the equations

$$\overset{s}{O}_{\alpha\beta} = \sum_{r}^{0\cdots s} p_{sr} \overset{r}{G}_{\alpha\beta} \qquad \text{with} \qquad p_{00} = 1 \ (s = 1 \ \ldots \ n_0)$$

and require that

$$\overset{s}{O}_{\alpha\beta} \overset{t}{G}^{\alpha\beta} = 0 \quad (t < s),$$

then if follows that

$$\overset{s}{O}_{\alpha\beta} \overset{t}{O}^{\alpha\beta} = 0 \quad (s \neq t),$$

which is the definition of an orthogonal system of tensors.

The $p_{sr}(s, r)$ must therefore satisfy the following system of equations:

$$\overset{r+t}{p_{sr}G} = 0 \; (s = 0 \ldots n_0 - 1, t = 0 \ldots s - 1).$$

In the nondegenerate case $\overset{r}{G} = s_r(r)$, and $\sum_{r}^{0-s} p_{rs}\lambda^r$ represents the sth Legendre polynomial defined for the finite range $(1 \ldots n)$.

It hardly seems advisable to use unitary systems, as irrationalities are thereby introduced.

Theorem (5, 3). Every rational tensor function of the $g_{\alpha\beta}$ and $G_{\alpha\beta}(\alpha, \beta)$ can be expressed uniquely in terms of a system of fundamental symmetric tensors.

In order to give the expansion of such a tensor function in the general case would require the introduction of a somewhat complicated notation. In order to avoid this, the theorem will be proved and the expansion given for tensors of the fourth order, with a few remarks concerning those of higher order. It will not be difficult then for the reader to extend the proof and expansion to tensors of higher order.

By theorem (4, 6) any rational tensor function of $g_{\alpha\beta}$ and $G_{\alpha\beta}$ (α, β) must be so expressible:

$$K_{\alpha\beta\gamma\delta} = \overset{\text{I}}{\sum_{a}^{f_a=1}} L_a g_{a,\,\alpha\beta} g_{a,\,\gamma\delta} + \overset{\text{II}}{\sum_{a}^{f_a>1}} M_a g_{a,\,\alpha\beta} g_{a,\,\gamma\delta}$$

$$+ \overset{\text{III}}{\sum_{a}^{f_a>1}} N_a g_{a,\,\alpha\gamma} g_{a,\,\beta\delta} + \overset{\text{IV}}{\sum_{a}^{f_a>1}} P_a g_{a,\,\alpha\delta} g_{a,\,\beta\gamma} + \overset{\text{V}}{\sum_{a\neq b}^{1-n_\delta}} c_a c_b$$

$$\times [\overset{\text{VI}}{Q_{a,\,b}} g_{a,\,\alpha\beta} g_{b,\,\gamma\delta} + \overset{\text{VII}}{R_{a,\,b}} g_{a,\,\alpha\gamma} g_{b,\,\beta\delta} + S_{a,\,b} g_{a,\,\alpha\delta} g_{b,\,\beta\gamma}].$$

Furthermore this representation is unique, that is the coefficients L_a, M_a, N_a, P_a, $Q_{a,\,b}$, $R_{a,\,b}$, and $S_{a,\,b}$ are uniquely determined by the tensor $K_{\alpha\beta\gamma\delta}$.—In order to show, this it is only necessary to remark that:

(1) The tensors represented by each of the various terms I, II + III + IV, V, VI, and VII are orthogonal to one an-

other. It follows that these tensors are uniquely determined by the $K_{\alpha\beta\gamma\delta}$.

(2) The tensors represented by the various elements under the summation signs in each of the terms I, V, VI, VII are mutually orthogonal. It follows that the coefficients L_a, $P_{a,\,b}$, $Q_{a,\,b}$, and $R_{a,\,b}$ are uniquely determined by the $K_{\alpha\beta\gamma\delta}$.

(3) From (1) it follows that the tensor represented by the terms II + III + IV = $U_{\alpha\beta\gamma\delta}$ is uniquely determined by the $K_{\alpha\beta\gamma\delta}$.

Compute:

$$\begin{aligned} U_{\alpha\beta\gamma\delta}g_a{}^{\alpha\beta}g_a{}^{\gamma\delta} &= M_a f_a{}^2 + N_a f_a + P_a f_a \\ U_{\alpha\beta\gamma\delta}g_a{}^{\alpha\gamma}g_a{}^{\beta\delta} &= M_a f_a + N_a f_a{}^2 + P_a f_a \\ U_{\alpha\beta\gamma\delta}g_a{}^{\alpha\delta}g_a{}^{\beta\gamma} &= M_a f_a + N_a f_a + P_a f_a{}^2. \end{aligned} \qquad (a)$$

It follows that M_a, N_a, P_a (a) are uniquely determined by the $U_{\alpha\beta\gamma\delta}$ and therefore also by the $K_{\alpha\beta\gamma\delta}$.

We shall now introduce a certain matrix

$$\Lambda^{(r,\,s)}_{(a,\,b)}\ (r,\,s = 1 \cdots \infty)(a,\,b = 1 \cdots n_0).^*$$

The rows of this matrix are represented by the ordered pairs (a, b) and the columns by the ordered pairs (r, s). The row s and columns will be considered ordered in some definite manner, while rows and columns corresponding to pairs of the type (a, a) and (r, r) will be omitted. This matrix has the rank $n_0(n_0 - 1)$. A particular square minor $\lambda^{(r,\,s)}_{(a,\,b)}$ of order and rank) $n_0(n_0 - 1)$ will be chosen, corresponding to a certain subset σ of the elements $(r - 1 \cdots \infty)$. The reciprocal of $\lambda^{(r,\,s)}_{(a,\,b)}$ will be denoted by $\mu_{(a,\,b),\,(r,\,s)}$.

The following relationships are made use of:

$$g_{a,\,\alpha\beta}g_{a,\,\gamma\delta} = \sum_{r}^{1-n_0} \mu_{a,\,r}\overset{r,\,1}{G}_{\alpha\beta\gamma\delta} \qquad (f_a = 1)$$

$$g_{a,\,\alpha\beta}g_{a,\,\gamma\delta} = \sum_{r}^{1-n_0} \mu_{a,\,r}\overset{r,\,>1}{G}_{\alpha\beta\gamma\delta} \qquad (f_a > 1)$$

$$e_a e_b g_{a,\,\alpha\beta}g_{b,\,\gamma\delta} = \sum_{r\neq s}^{\sigma} (\overset{r}{G}_{\alpha\beta}\overset{s}{G}_{\gamma\delta} - \overset{r+s}{G}_{\alpha\beta\gamma\delta})\mu_{(a,\,b),\,(r,\,s)} \qquad (a \neq b)$$

$^*\Lambda^{(r,\,s)}_{(a,\,b)} = \lambda^r_a\lambda^s_b$.

where

$$\overset{r,\,1}{G}_{\alpha\beta\gamma\delta} = \sum_{a}^{f_a=1} \lambda_a{}^r g_{a,\,\alpha\beta} g_{a,\,\gamma\delta} \qquad (r)$$

$$\overset{r,\,>1}{G}_{\alpha\beta\gamma\delta} = \sum_{a}^{f_a>1} \lambda_a{}^r g_{a,\,\alpha\beta} g_{a,\,\gamma\delta}. \qquad (r)$$

Substituting these in the previous expansion for $K_{\alpha\beta\gamma\delta}$, we obtain:

$$K_{\alpha\beta\gamma\delta} = \sum_{r}^{1-n_0} [A_r \overset{r,\,1}{G}_{\alpha\beta\gamma\delta} + B_r \overset{r,\,>1}{G}_{\alpha\beta\gamma\delta} + C_r \overset{r,\,>1}{G}_{\alpha\gamma,\,\beta\delta} + D_r \overset{r,\,>1}{G}_{\alpha\beta,\,\gamma\delta}]$$

$$+ \sum_{r\neq s}^{\sigma} [A_{r,\,s}(\overset{r}{G}_{\alpha\beta}\overset{s}{G}_{\gamma\delta} - \overset{r+s}{G}_{\alpha\beta\gamma\delta}) + B_{r,\,s}(\overset{r}{G}_{\alpha\gamma}\overset{s}{G}_{\beta\delta} - \overset{r+s}{G}_{\alpha\gamma,\,\beta\delta}) + G_{r,\,s}(\overset{r}{G}_{\alpha\delta}\overset{s}{G}_{\beta\gamma} - \overset{r+s}{G}_{\alpha\delta\beta\gamma})]$$

which is a unique representation of $K_{\alpha\beta\gamma\delta}$ in terms of the fundamental symmetric tensors:

$$\overset{r,\,1}{G}_{\alpha\beta\gamma\delta};\quad \overset{r,\,>1}{G}_{\alpha\beta\gamma\delta} \qquad (r = 1 - n_0);$$

$$(\overset{r}{G}_{\alpha\beta}\overset{s}{G}_{\gamma\delta} - \overset{r+s}{G}_{\alpha\beta\gamma\delta}) \qquad (r^\sigma,\ s^\sigma)$$

and the $s_{f,\,r}$. The symmetry character of the invariants A_a etc., follows from the uniqueness of the representation and the symmetry character of the $K_{\alpha\beta\gamma\delta}$.

In the same way it can be shown that every rational tensor function of the sixth order can be expressed as a linear combination with invariant coefficients of the following system of fundamental symmetric tensors:

$$\overset{r,\,1}{G}_{\alpha\beta\gamma\delta\epsilon\eta};\quad \overset{r,\,>1}{G}_{\alpha\beta\gamma\delta\epsilon\eta};\quad \overset{r,\,1}{G}_{\alpha\beta\gamma\delta}\overset{s}{G}_{\epsilon\eta} - \overset{r+s,\,1}{G}_{\alpha\beta\gamma\delta\epsilon\eta};\quad \overset{r,\,>1}{G}_{\alpha\beta\gamma\delta}\overset{s}{G}_{\epsilon\eta} - \overset{r+s,\,>1}{G}_{\alpha\beta\gamma\delta\epsilon\eta}$$

$$(r,\ s = 1 \cdots n_0)$$

$$\overset{r}{G}_{\alpha\beta}\overset{s}{G}_{\gamma\delta}\overset{t}{G}_{\epsilon\eta} - \overset{r+s}{G}_{\alpha\beta\gamma\delta}\overset{t}{G}_{\epsilon\eta} - \overset{r+t}{G}_{\alpha\beta\epsilon\eta}\overset{s}{G}_{\beta\gamma} - \overset{s+t}{G}_{\gamma\delta\epsilon\eta}\overset{r}{G}_{\alpha\beta} - \overset{r+s+t}{G}_{\alpha\beta\gamma\delta\epsilon\eta}$$

$$(r^\sigma,\ s^\sigma,\ r^\sigma)$$

together with all the different tensors produced from these by interchanges of the $\alpha\beta\gamma\delta\epsilon\eta$.

In this case it is necessary to consider a matrix $\Lambda^{(r,\,s,\,t)}_{(a\ b\ c)}$ defined analogously to the $\Lambda^{(r,\,s)}_{(a,\,b)}$ which is at most of rank $n_0(n_0 - 1)(n_0 - 2)$. It probably has this rank. However,

it is not necessary to prove this. It is only necessary to choose for the $\lambda^{(r,s,t)}_{(a,b,c)}$ a minor of maximum rank, whose determinant is not zero, corresponding to some particular subset σ of the elements $r = 1 \cdots \infty$.

The theory of adjoint fundamental systems of symmetric tensors of the second order as well as the theory of completely orthogonal systems of such tensors, which have been discussed after theorem (5, 2) can be easily extended to tensors of higher order.

In another paper, the rational tensor functions of $g_{\alpha\beta}$ and $G_{\alpha\beta}$ and their first derivatives will be studied by the methods which have been developed in the present paper. In conclusion, the writer wishes to thank Professor Pauli and Professor von der Waerden for helpful suggestions and criticisms.

AN INVESTIGATION OF THE SHENSTONE EFFECT.

BY

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INTRODUCTION.

Since 1887 when it was first shown that a negatively charged metal would emit electrons when illuminated by light of proper frequency, the photoelectric effect has been the subject of much investigation. Despite the time and effort devoted to the study of photoelectric phenomena there had existed, until quite recently, much uncertainty concerning fundamental questions in photoelectricity. One important cause of this uncertainty was the failure on the part of workers in this field to determine the true effect of gases, naturally occluded by the metal surfaces, upon the photoelectric emission of these surfaces. It was, for example, concluded at one time that the presence of gas in a metallic surface was essential to the process of photoelectric emission. At other times, effects that were at first sight apparently new, were ultimately traced to occluded gases. The present article is a report on an experimental study of the "Shenstone effect," an effect intimately dependent, as will be shown, on the gases present in the photoelectrically active metal.

In a paper [1] titled "The Effect of an Electric Current on the Photo-Electric Effect" A G. Shenstone reported in 1921 that the photoelectric sensitivity of cast bismuth plates was increased up to 100 per cent. by the passage of a direct current of several amperes through them. This effect has been referred to in the literature as the "Shenstone effect." To explain the increased emission which he had observed Shenstone advanced two hypotheses: 1. that the effect might be a temperature effect; and 2. that it might be connected with orientation of the elementary crystals of the metal.

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[1] A. G. Shenstone, *Phil. Mag.*, 41, 916 (1921).

In 1923 Shenstone presented evidence eliminating the second of these hypotheses [2] and also evidence purporting to eliminate the first. He furthermore [2, 3] eliminated the variation of the gas content of the metal, or of the auxiliary apparatus, as a cause of the effect observed and he was, therefore, finally driven to the conclusion that the changes in photoelectric sensitivity are to be attributed in some way to the current itself passing through the metal.

It should be stated that there exists amongst commentators considerable uncertainty, not merely as to the causes of this effect, but even as to its existence,[4] and the present series of experiments were initiated to make a systematic investigation of the subject.

To this end extensive preliminary experiments were made with bismuth which proved that the gas occluded by the metal emitting the photoelectrons was a factor of very considerable importance in influencing the photoelectric emission. Large changes in the photoelectric sensitivity were observed in the course of progressive degassing of each specimen of bismuth. Shenstone's curves for bismuth, it is to be remarked, were obtained with fresh specimens newly introduced into the photoelectric cell and consequently threw no light on this point. It is obvious that bismuth because of its low melting point is a peculiarly difficult metal to degas even incompletely. Further experiments were therefore carried out employing a metal (molybdenum) and a design of photoelectric cell with which good outgassing was possible. These new experiments are reported on in the present paper.

DESIGN OF PHOTOELECTRIC CELL. AUXILIARY APPARATUS.

Two different types of apparatus were used; designated in what follows as the "first photoelectric cell" and the "second photoelectric cell." The second, and more unique, cell will be first described.

This "second photoelectric cell" was constructed entirely of pyrex with a fused quartz window joined to the main tube with a quartz-to-pyrex graded seal. Waxed joints were

[2] A. G. Shenstone, *Phil. Mag.*, **45**, 918 (1923).
[3] A. G. Shenstone, *Phil. Mag.*, **42**, 596 (1921).
[4] Hughes and DuBridge, "Photoelectric Phenomena," p. 501.

thus entirely eliminated. Fig. 1 gives the details of construction. *W* was a flat, polished plate of fused quartz having a clear central area of 1/2 inch diameter. *B*, the source of photoelectrons, was a strip of molybdenum foil 13 mm × 6 mm × 0.001 inch. This metal * had, according to the manufacturer, a purity of better than 99.8 per cent. *A* was a conically shaped anode of copper. A thin circular sheet of nickel, not shown in the figure, was attached by means of a simple hinge to the open end of the anode. When this flap was raised into position by means of an electromagnet, the end of the anode toward the quartz window was covered and condensation of material on the window during outgassing of the molybdenum was thus prevented. *C* was a heating grid of 11 mil tungsten wire bent to the form of a rake and supported about 0.5 mm from the rear face of the molybdenum foil so as to be completely hidden from the light brought through the quartz window. The wires *T*, of chromel and alumel (B. & S. gauge 28), were joined in a spot-weld to the rear face of the molybdenum, and formed the hot junction of a thermocouple. As shown in Fig. 1 the other ends of the chromel and alumel wires were spot-welded to tungsten wires, *L*, sealed directly into the reëntrant pyrex tube.

The chromel-alumel thermocouple used in these experiments contained an intermediate metal, tungsten, which was used to bring the thermocouple wires through the pyrex in a direct glass to metal seal. To keep the two ends of these intermediate pieces of tungsten at the same temperature a water cooling system was used and, in addition, the tungsten-to-pyrex seals were placed in the lower part of the photoelectric cell as far removed as possible from the hot junction of the thermocouple.

The photoelectric cell was sealed directly to a mercury diffusion pump with an intermediate stopcock. To prevent stopcock grease vapors from diffusing back into the photoelectric cell a trap, located between the photoelectric tube and the stopcock, was kept permanently packed with a slush of dry ice and acetone at a temperature of about $-72°$ C.

The "first photoelectric cell" was similar in construction to that just described. Two waxed joints were, however,

* Obtained from the Callite Products Company.

included in this first cell; one to fasten a crystalline quartz window to the pyrex tube, the other to make vacuum-tight a ground glass joint through which the wires of a copper-constantan thermocouple were brought.

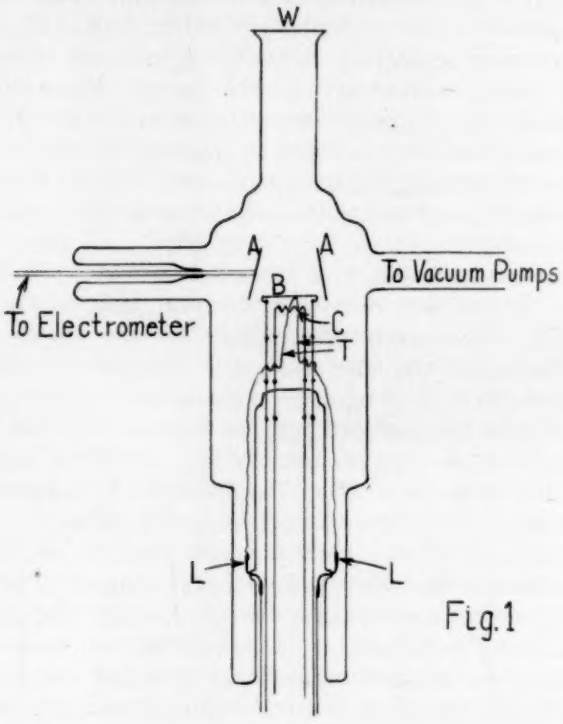


Fig. 1

Figure 2 shows the electrical connections to the photoelectric cell, corresponding letters in Figs. 1 and 2 indicating corresponding parts. The photoelectric currents were measured with a Compton electrometer adjusted to a sensitivity of 3280 mm/volt with a needle potential of 143 volts. The electrometer was shunted across an S. S. White resistor, R, of 5.5×10^9 ohms which gave with this voltage sensitivity, a current sensitivity of 5.5×10^{-14} amperes/mm. The molybdenum was maintained at a negative potential (usually -12 volts) by the battery H, a potential divider, P, being used to keep constant the potential drop between the molybdenum

and ground. The temperature of the molybdenum was measured by carrying the thermocouple leads, *T*, to a cold junction at 0° C., and thence to an L. & N. potentiometer calibrated directly in Centigrade degrees.

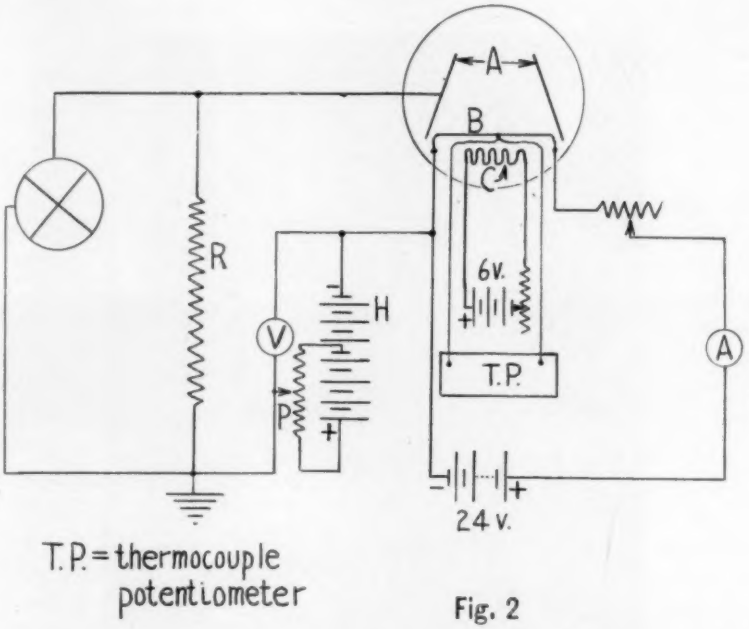
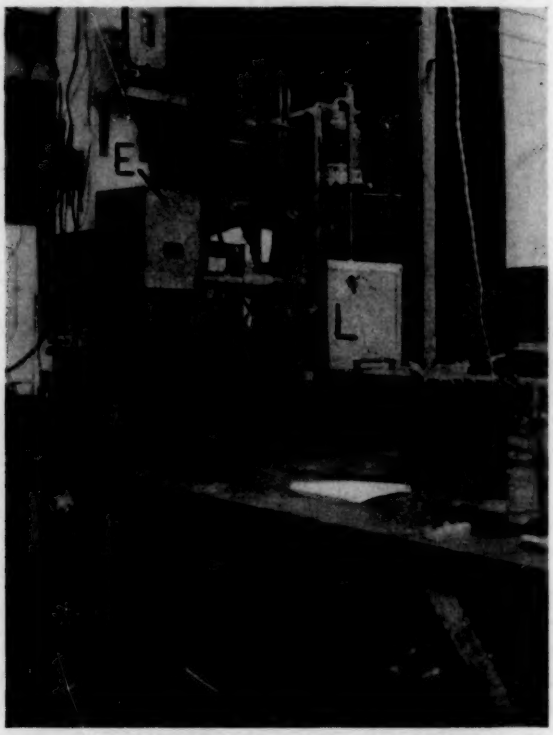


Fig. 2

The photoelectric cell and the electrometer together with its leads, were carefully shielded. The entire photoelectric tube was wrapped with sheets of lead foil held in place by many turns of fine copper wire connected to ground. In addition to this shielding a sectional metal box completely enclosed the photoelectric cell. This box was joined to a similar box, housing the electrometer, by a brass tube surrounding the electrometer lead. This double shielding of the photoelectric tube proved very effective.

The source of illumination employed to stimulate the photoelectric emission was a quartz mercury lamp operated with a current of 3.1 amperes at a potential of 39 volts across the arc terminals. The current, adjusted continuously to 3.1 amperes by a group of rheostats in series with the arc

terminals, was supplied by 50 large capacity lead plate storage batteries connected in series. This arrangement produced a source of radiation of very constant intensity. The total radiation from this lamp was incident upon the molybdenum after passing through a collimating tube and a



Arrangement of the various parts of the apparatus. *E*, electrometer housing; *L*, sectional shield enclosing photoelectric cell; *B*, box containing quartz mercury lamp; *P*, mercury pumps.

series of diaphragms. In adjusting these diaphragms the nickel flap, previously described, was raised into position in front of the molybdenum and used as a screen upon which the outline of the rectangular patch of light introduced into the photoelectric cell readily could be seen.

The copper anode was baked out in a separate vacuum furnace just before introduction into the photoelectric cell. The molybdenum specimens were carefully cleaned with fine

sandpaper and then washed in alcohol and distilled water before introducing them into the tube. All glass parts of the photoelectric cells were thoroughly washed with alcohol and distilled water before finally sealing the cells to the vacuum pumps. The pumps were run continuously while taking observations, a "flat" vacuum being always maintained. The pressures attained were thus beyond the limit of measurement of the McLeod gauge which could be read to 10^{-6} mm.

EXPERIMENTAL PROCEDURE.

The experiments on the Shenstone effect in molybdenum are divided into three groups corresponding to these three stages of outgassing: (1) no outgassing; (2) partial outgassing; (3) good outgassing. The experiments under the first two of these divisions were carried out with the use of the first photoelectric cell while those under the last heading could be conducted only with the second photoelectric cell.

With the first photoelectric cell, containing the two waxed joints, a limited amount of outgassing only was possible. The sections of the tube around the waxed joints were wrapped with several turns of lead tubing through which cold water was circulated. The remaining glass parts of the tube were heated with a Bunsen burner to such temperatures as caused the wax to soften. The "no outgassing" experiments were then performed. After this, the molybdenum was partially outgassed by passing heating current up to 10 amperes, corresponding to about 650° C., through the metal. The exact number of hours of this "partial outgassing" is given below along with the actual experimental results. The experiments listed under "partial outgassing" were then conducted. The "no outgassing" and "partial outgassing" experiments were performed with the same molybdenum specimen. In the last group of experiments, titled "good outgassing," use was made of the second photoelectric cell. The entire cell was degassed at temperatures up to 507° C., by the use of electrical heating coils placed around sections of the tube. Those parts not degassed with the heating coils were thoroughly torched with a Bunsen flame. This heat treatment was continued until the McLeod gauge registered

a "flat" vacuum, the pressure being measured while the degassing was in progress. This usually required about six hours and did not involve any direct outgassing of the molybdenum specimen. A series of experiments on the Shenstone effect were then carried out to compare the results with the "no outgassing" experiments conducted with the first photoelectric cell. Then, the temperature of the molybdenum was raised to near the melting point of the chromel-alumel thermocouple by using an electric current. Results were obtained for two molybdenum specimens. One was heated at discontinuous intervals for $32\frac{1}{4}$ hours at temperatures up to 1188° C., the other was heated in the same manner at 1225° C. for 23 hours. The temperatures were measured with an optical pyrometer made by the Scientific Materials Co. of Pittsburgh, Penna. The thermocouple was not used to measure these temperatures because the conduction of heat away from the hot junction at these high temperatures prevented an accurate measurement of temperature by this means. The tungsten grid behind the molybdenum was outgassed at the same temperatures for approximately the same length of time.

Three types of experiments were conducted:

Type I: Temperature of molybdenum variable, currents through molybdenum variable (similar to Shenstone's experiments). In these experiments the following procedure was used. At the beginning of a run the molybdenum specimen was illuminated and no current was passed through the specimen until the photoelectric current attained a steady value. With specimens that had as yet been given no direct outgassing this "fatiguing" process usually required several hours. Currents up to 5 amperes were then passed through the metal and sufficient time was allowed, in the case of each current, for the change in photoelectric sensitivity to take place. The attained steady values of the photoelectric current were measured while the current was passing through the metal so that the corresponding steady value of the temperature could be measured. The potential difference between the molybdenum and the copper anode was maintained constant by adjustment of the potential divider (P, in Fig. 2). In most cases readings of the photoelectric current

also were taken with the current through the metal interrupted, to determine any possible effect of the magnetic field on the emitted photoelectrons. The difference between the readings taken with the current flowing and interrupted was at most 2 per cent. During the course of a run the molybdenum specimen was constantly exposed to the radiation from the quartz mercury arc. The time intervals that were allowed between changes of the current through the metal during the first run on a specimen likewise were allowed between corresponding readings in all subsequent runs on the same specimen.

Type II: Temperature of molybdenum variable; no currents through molybdenum. To determine the effect of temperature, runs were taken under conditions in which temperature changes in the molybdenum were produced indirectly, without the passage of a current through the metal. This was accomplished by passing currents through the tungsten heating grid located behind the molybdenum, keeping all other factors the same. Approximately the same temperature changes were produced by this heating grid as were caused by the current passing directly through the specimen. In making runs of this kind the heater current was always interrupted while taking the electrometer reading in order to avoid possible distortions, due to the heater current, in the electrostatic field between the molybdenum and the collecting anode. The thermocouple readings were taken simultaneously with the electrometer readings.

Type III: Temperature of molybdenum constant; current through molybdenum variable. To obtain further information about the effect of temperature on the Shenstone effect, "constant temperature" runs were made. That is, the molybdenum specimen was maintained at a constant temperature while varying currents were passed through it. This was accomplished in the following manner. By means of a heating current through the tungsten grid the temperature of the molybdenum was raised to a fixed value. A small current was then passed through the molybdenum which normally would raise the temperature of the metal. To keep the temperature at the fixed chosen value the tungsten heater current was decreased the proper amount. As the currents through the molybdenum were gradually increased, the tung-

sten heater currents were appropriately decreased. Thus, by matching the current through the molybdenum against the current through the tungsten grid, the molybdenum was held at a fixed temperature even though currents passed through it. The thermocouple was used to check the constancy of this temperature. All other factors were kept the same during these constant temperature runs. The currents through the molybdenum and through the tungsten heater were interrupted during the reading of the photoelectric current to eliminate possible distortions in the electrostatic field as explained above. The thermocouple readings were taken simultaneously with the electrometer readings.

RESULTS.

A total of 46 runs on 5 specimens of molybdenum were made. These runs are divided, as stated above, into three groups corresponding to three stages of outgassing. A careful and progressive study of the effect of occluded gas on the photoelectric sensitivity thereby was made.

1. *No outgassing.* (Figs. 3 and 4.) The results grouped under "no outgassing" were obtained with the first photoelectric cell containing two waxed joints. Two molybdenum specimens were used, with identical results. Fig. 3 obtained with the second specimen gives a typical set of curves illustrating the variation of photoelectric current with the current through the molybdenum (Type 1 experiments). The ordinates of this curve, and of all succeeding curves, are electrometer deflections from zero. The first of these curves, number 1 of Fig. 3, was obtained with the fresh molybdenum specimen which had received no direct outgassing, the first point (*D*) on curve 1 corresponding to the photoelectric current taken 5 hours after introducing the specimen into the photoelectric cell. Curve number 2 is the return curve obtained by decreasing the currents through the molybdenum. Curves 1 and 2 were recorded in one continuous run of 8 hours. The large increase in photoelectric sensitivity, 260 per cent., exhibited by curve 1 and the lack of reproducibility between curves 1 and 2 are to be particularly noted. Curves 3 and 4 of Fig. 3 are the result of the second run. The specimen stood in vacuum, cut off from the pumps, for a

period of 38 hours between the first and second runs. The conditions (electrometer sensitivity, intensity of illumination,

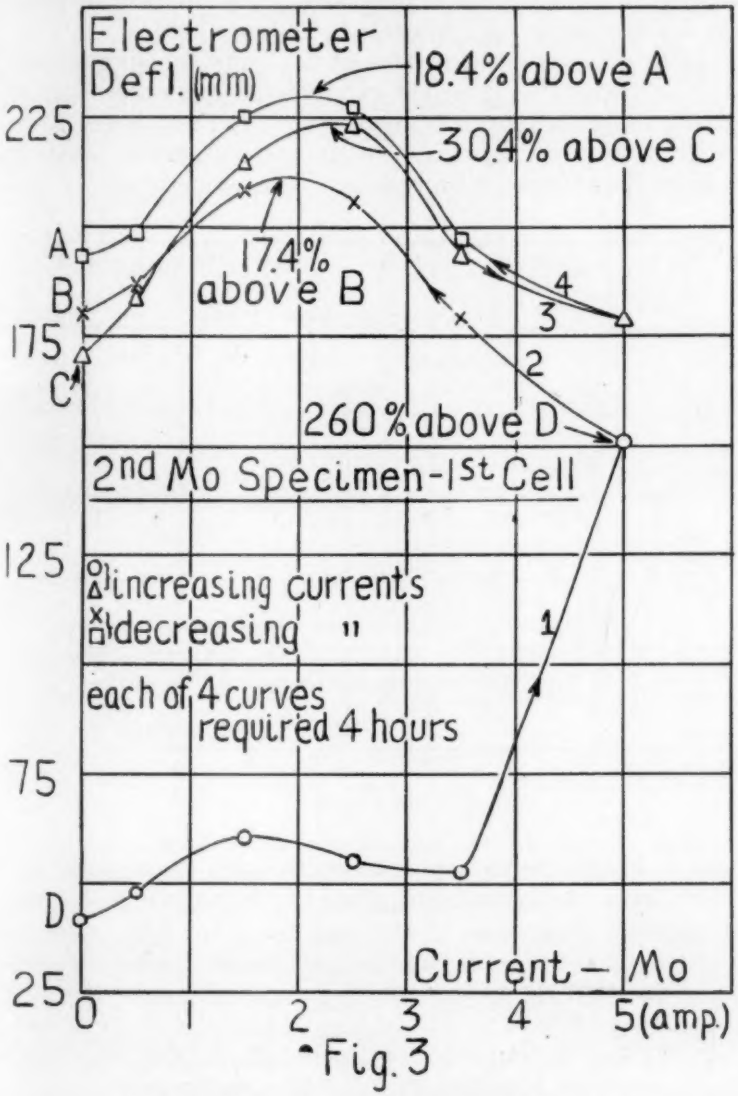


Fig. 3

etc.) were identical for the two runs. The third run, not given here, exhibited the same characteristics as the second

run except that the maximum in the vicinity of 2 amperes had dropped to 28.8 per cent. With the first specimen the same cycle of changes was observed; at the end of the fourth run the maximum had dropped to 19.6 per cent.

Figure 4 gives the results of a "constant temperature, variable current" (Type III experiment) run made with the second molybdenum specimen. The method used in determining this curve is given in the last paragraph under the heading Experimental Procedure. The abscissæ of Fig. 4 are

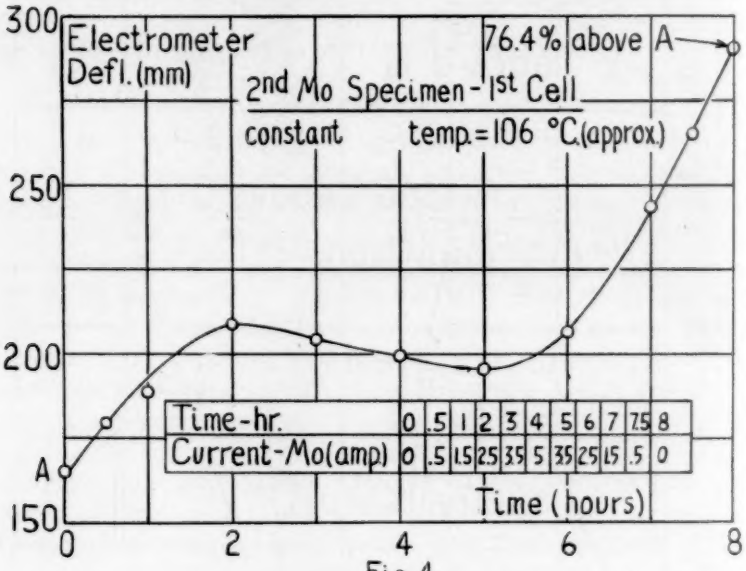
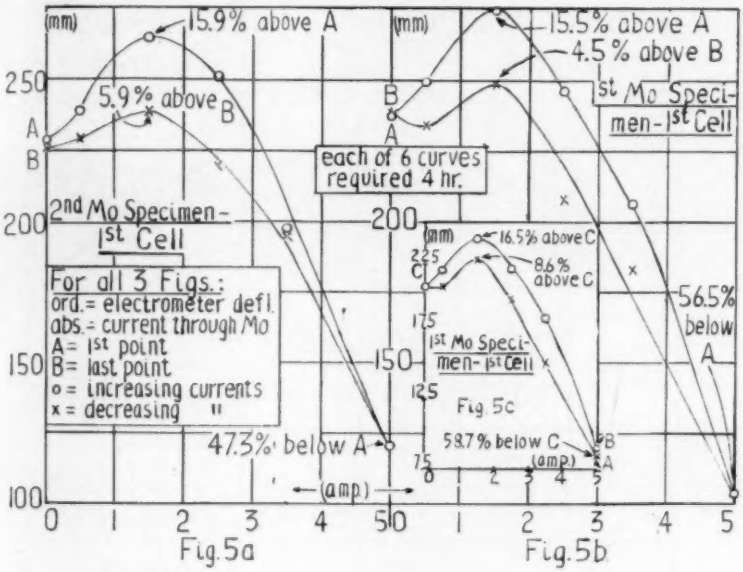


Fig. 4

times in hours, the currents passed through the molybdenum in obtaining each point being given by the table included with the figure. The value of the resistance (*R*, Fig. 2) in the electrometer circuit was changed just before taking the curve of Fig. 4 so that the ordinates of this curve cannot be compared with those of Fig. 3.

2. *Partial Outgassing.* (Figs. 5*a*, 5*b*, 5*c* and 6.) These curves were also obtained with the first photoelectric cell and show the effect of "partial outgassing" on the variation of the photoelectric emission of molybdenum with current

through the metal. The molybdenum specimen was heated to about 650° C. with currents between 7 and 10 amperes depending on the specimen. Fig. 5*a* gives the first run taken with the second specimen after $6\frac{1}{4}$ hours of outgassing with heating currents between 9 and 10 amperes. Fig. 5*b* gives the result of the corresponding run taken with the first molybdenum specimen after $2\frac{3}{4}$ hours of outgassing with heating currents of 7.3–8.0 amperes. The close agreement of the curves is obvious. It is to be noted that the return curves



are in each case below the curves for increasing currents. Runs were also taken with the order of passing currents through the molybdenum reversed, beginning with a high current instead of zero current, the currents being first decreased and then increased. Fig. 5*c*, the inset on Fig. 5*b*, gives the results of one such run, and shows that the curve for decreasing currents is, despite the reversal in the order of taking the observations, still below the curve for increasing currents. The percentage variations shown on the curve of Fig. 5*c* are also comparable with those of Figs. 5*a* and 5*b*. Runs were also taken to determine the effect of varying the

time of the run. Each pair of curves of Figs. 5*a*, 5*b* and 5*c* required a total time of 8 hours. Runs requiring a total time of 2½ hours, with currents changed every 15 minutes, gave curves of the same nature.

Inspection of Figs. 5*a*, 5*b* and 5*c* indicates a maximum at about 1.5 amperes for the curves of increasing currents. To investigate more accurately the variation of the photoelectric current in the neighborhood of this maximum the

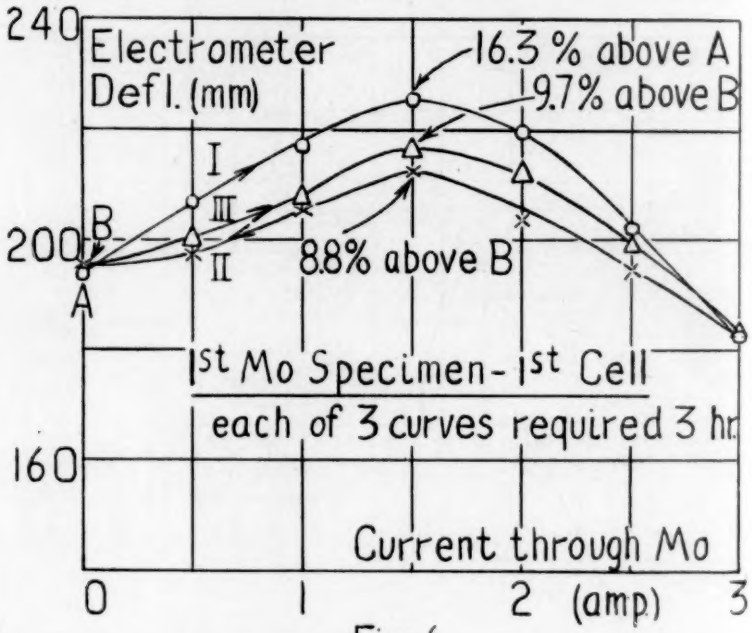


Fig. 6

curves of Fig. 6 were obtained. These curves show the effect of making three consecutive excursions through the region of maximum photoelectric current.

The "partial outgassing" groups of curves are characterized by a further decrease in the height of the maximum from that observed on the "no outgassing" curves, and by the appearance of a definite decrease in emission with increasing currents through the specimen. Thus a 5 ampere current, which initially caused an increase over 200 per cent.,

here (Figs. 5*a*, 5*b* and 5*c*) produced a decrease of about 50 per cent. The degree of reproducibility is also better than in the "no outgassing" curves as is best seen by reference to Fig. 6.

3. *Good Outgassing.* (Figs. 7 and 8.) The results presented here were obtained with the use of the second photoelectric cell previously described (Fig. 1). After carefully outgassing the entire photoelectric cell as described under Experimental Procedure, but before directly outgassing the

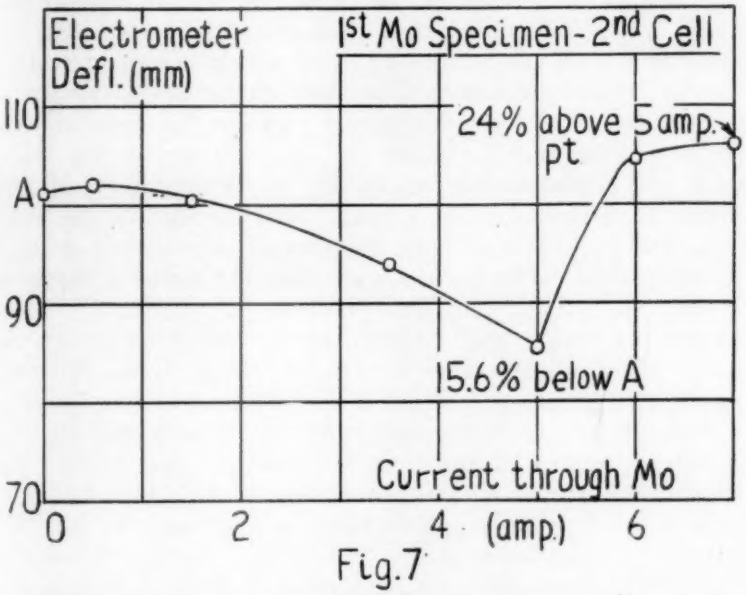


Fig. 7

molybdenum specimen, several runs of the kind made with the first photoelectric cell (given in Fig. 3) were recorded. The curves obtained were of the same nature as those of Fig. 3. The fourth run of this series is given in Fig. 7. It is to be noted that there is a steady decrease in the photoelectric current up to 5 amperes; preceding runs having however exhibited the characteristic maxima shown by Fig. 3. Also, there is a sudden increase in the photoelectric emission beyond 5 amperes. It is important to record that in the run of Fig. 7 currents greater than 5 amperes were, for the first time, passed through the molybdenum specimen. Fig. 7 is

included under the heading "Good Outgassing" because this curve was obtained with the second photoelectric cell.

Following this preliminary series of runs a new molybdenum specimen was inserted in the photoelectric cell; the first foil having burned out during outgassing. This second specimen was given an outgassing treatment of $32\frac{1}{4}$ hours at temperatures up to 1188° C. During the course of this outgassing the three types of experiments, explained under the heading Experimental Procedure, were performed. These experiments constitute the clinching evidence in the present study. The results, obtained after $26\frac{3}{4}$ hours outgassing, are given in Fig. 8, the four curves of which comprise a unit. The "*a*" curve gives the variation of photoelectric current as a function of the currents passed through the molybdenum (Type I experiment). In the "*b*" and "*c*" curves, the effects upon the photoelectric sensitivity of temperature change produced directly by the passage of a current through the metal and indirectly without the passage of a current through the molybdenum, are compared. The "*b*" curve is the same as the "*a*" curve with the photoelectric currents plotted against the temperature, instead of the currents through the metal, as abscissæ. The "*c*" curve (Type II experiment) illustrates the change produced in the photoelectric emission of molybdenum as the temperature of the molybdenum was varied by means of the tungsten heating grid behind the metal, no currents being passed through the molybdenum. As nearly as possible, the temperature changes of the "*b*" curve were reproduced in the "*c*" curve. In the "*d*" curves (Type III experiment) the temperature of the molybdenum was maintained nearly constant while currents up to 2.8 amperes were passed through the molybdenum. The method of doing this has been explained above. In the three types of experiments, illustrated by the curves of Fig. 8, great care was exercised to keep all factors, except those being studied, constant. The three experiments were thus performed in immediate succession, over 16 hours in all being required. The same time interval of 1/2 hour was allowed between corresponding readings.

In all, three sets of runs of the type of Fig. 8 were made with the second molybdenum specimen used in the second

photoelectric cell. The other two sets of runs, taken after approximately 7 and 19 hours of outgassing, exhibited the

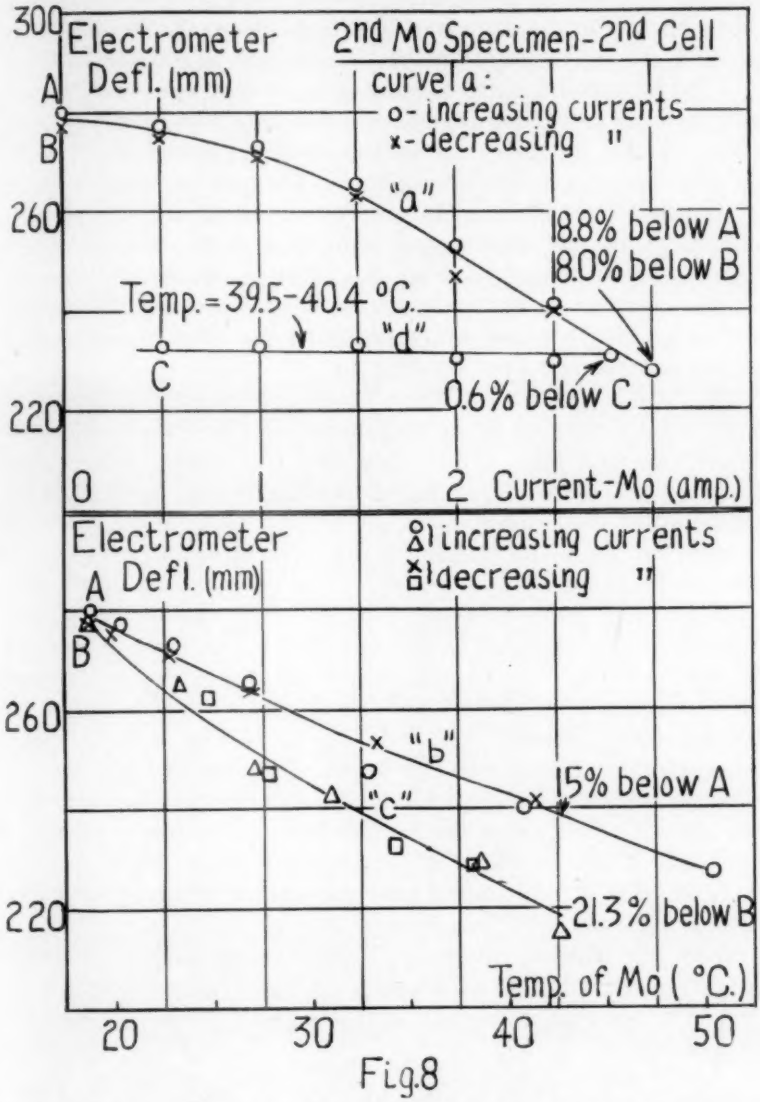


Fig.8

same characteristics as those in Fig. 8 with the important exception that the curves for increasing and decreasing

currents and for increasing and decreasing temperatures do not agree as closely as in Fig. 8. The curves become more nearly reproducible as the time of outgassing increases. After obtaining the curves of Fig. 8, 5½ hours of outgassing at 1188° C. failed to improve the high degree of reproducibility.

The important series of runs given in Fig. 8 were repeated with a third molybdenum specimen of about the same dimensions as the second. As stated under Experimental Procedure this specimen was given an outgassing treatment of 23 hours at 1225° C. The curves obtained with this third specimen, are not reproduced here since they are practically identical with those of Fig. 8. With both specimens the photoelectric current, as measured from time to time during the outgassing process, increased with the time of outgassing in the manner reported by Martin.[5]

Several remarks on this "good outgassing" group of experiments may be made to summarize the results obtained under this heading. The outgassing treatment at temperatures around 1200° C. changed essentially the nature of the variation of the photoelectric sensitivity as a function of the current through the specimen, for after this outgassing the photoelectric current decreased with an increase of current through the molybdenum and the previously observed maximum has completely disappeared. The close similarity of the "*b*" and "*c*" curves shows the direct dependence of the variation of photoelectric sensitivity of molybdenum on temperature change. The "*d*" curve of Fig. 8 shows that the variation of photoelectric sensitivity observed in the "*a*" curve is practically independent of the current passed through the molybdenum when the temperature of the metal remains unchanged, for in the "*d*" curve the temperature of the molybdenum is maintained nearly constant while currents up to 2.8 amperes are passed through the metal. Fig. 4 is a similar "constant temperature" curve taken with a specimen of molybdenum before outgassing. Comparison of Fig. 4 with curve "*d*" of Fig. 8 clearly illustrates the difference caused by outgassing.

After performing the long series of experiments represented by Figs. 3 to 8 in this report, a test to determine the

[5] M. J. Martin, *Phys. Rev.*, 33, 991 (1929).

effect of admitting air to the photoelectric cell was made. A specimen was given $25\frac{1}{4}$ hours of outgassing at 1225° C. The photoelectric current, as measured by the electrometer deflection, was then 204.3 mm. Air at atmospheric pressure was then admitted to the photoelectric cell. After a period of 11 hours the photoelectric cell was pumped down to a "flat" vacuum and the photoelectric current was measured and found to be 43.0 mm, the ratio of the change in photoelectric current to this low value being 375 per cent. Fifteen minutes of heating at approximately 612° C. caused an increase in the deflection from 43.0 mm to 133.0 mm. In the preliminary work with bismuth these same characteristic features were observed.

DISCUSSION.

The changes in photoelectric sensitivity observed must, in general, be attributed to one or more of the following causes: (1) changes in the gas content of the metal emitting the photoelectrons; (2) temperature; (3) some effect of the current passed through the metal independent of its heating effect. With respect to these possibilities the present series of experiments establishes the following three points.

I. *The initial large increases in the photoelectric emission of molybdenum with the passage of currents through the metal must be definitely attributed to the removal of occluded gases.* A study of the results presented under the category of "no outgassing" proves that the initial increases of several hundred per cent. in the sensitivity, such as is typically shown by curve 1 of Fig. 3, must be due to the removal of gas from the molybdenum. Shenstone [1] and the author in some preliminary work observed the same large increases with fresh specimens of bismuth. In all cases these first runs were marked by considerable time lags in reaching equilibrium photoelectric currents and also by persistence of the effect after the currents through the metal had been interrupted; characteristics associated with the removal of gases from the metal. Further, and more direct, evidence toward this conclusion is found in the total failure of the return curve (2 of Fig. 3) to match the curve for increasing currents and by the fact that, after several runs, a decrease in the photoelectric current to its original low value could be produced by the simple procedure of intro-

ducing air at atmospheric pressure into the photoelectric cell. Shenstone, incidentally, presents no return curves in his papers. The curve of Fig. 7 is also very strong evidence of this effect of occluded gas. This curve is the fourth run on a molybdenum specimen. In the three preceding runs no currents greater than 5 amperes were passed through the molybdenum. In the curve of Fig. 7 currents greater than 5 amperes caused an immediate increase in the photoelectric current due to gas being liberated by the additional increase in temperature.

II. *The variation of the photoelectric emission of molybdenum with currents through the metal* (The Shenstone Effect) *is not due to some effect of the current independent of its heating effect.* In his second paper [2] Shenstone leans toward the conclusion that the changes in sensitivity he observed were due, in some way, to the current itself passing through the metal. Curves like the "*d*" curve of Fig. 8 are direct and apparently clinching evidence that the true cause of the effect is not some effect of the current independent of its heating effect. Reference to this curve shows that the photoelectric current was maintained practically constant for a period of three hours or more while currents up to 2.8 amperes were passed through the molybdenum. In all other repetitions of this curve the photoelectric current remained constant within 3 per cent. at most. The "constant temperature" curve of Fig. 4 exhibits a gradual increase of the photoelectric current as the currents through the molybdenum were increased to 5 amperes and then decreased to zero. This curve, obtained with a specimen which had received no definite outgassing at high temperatures, has its obvious explanation in the gradual removal of occluded gases. Shenstone's "constant temperature" curve for a copper film,[2] which curve is similar to our Fig. 4, may be explained in the same way.

III. *The change in photoelectric current with a current through the molybdenum is due to temperature change, either alone or with an associated gas effect.* The "*b*" and "*c*" pair of curves of Fig. 8 are definite proof of this conclusion. In the "*c*" curve, approximately the same variation in the photoelectric current, as is produced in the "*b*" curve by the passage of currents through the metal, is obtained by varying

the temperature of the molybdenum by heat radiation from a tungsten heater behind it and *without* the use of currents through the molybdenum itself. Comparison of the "*b*" with the "*c*" curves shows that the nature of the variation of the photoelectric current is the same in each case. The "constant temperature" curve just discussed (the "*d*" curve of Fig. 8) is added confirmation of the dependence of the observed changes of photoelectric sensitivity on temperature change. In this curve, involving variation in both the current through the molybdenum and the heater current, a constant temperature means a nearly constant photoelectric sensitivity. Shenstone [2] presents experimental evidence which he interprets as indicating that the observed increases in the photoelectric sensitivity of bismuth are independent of temperature. A bismuth plate was cemented to the open end of a brass tube which was waxed into the ground-glass joint of his photoelectric tube. Using no currents through the bismuth, he observed only a 2 per cent. variation in the photoelectric emission of the metal when it was heated from 15 to 50° C. by playing streams of warm water into the brass tube on the rear face of the bismuth plate. It must be pointed out that in this experiment the conditions were essentially different from those under which the large increases in photoelectric emission of bismuth were obtained by passing currents through the metal. Since in this case one face only of the plate was in vacuum, it is very likely that gases could be supplied to the bismuth from the exposed face as quickly as they were removed. The De Khotinsky cement used in fastening the plate to the brass tube would also provide a source of contamination.

Any attempt made to correlate the large amount of data collected in this series of experiments into a unifying theory must begin with those experiments grouped under the heading of "good outgassing" for in these the troublesome effect of gas contamination is minimized. Considering, then, the "*a*" curve of Fig. 8, the variation of photoelectric sensitivity with the current through the molybdenum may be said to be exactly reproducible. Furthermore, this curve agrees in general shape with that published by Martin [5] on the temperature effect of the photoelectric sensitivity of molybdenum.

Also, the time lags observed in the first runs on a specimen are here practically absent; the photoelectric current changes taking place as quickly as the temperature changes. It would therefore be very tempting to attribute the observed variations in photoelectric sensitivity to a pure temperature variation of the photoelectric effect. Against this view there is, however, theoretical and experimental evidence:

1. The energy distribution curve for the electrons of a metal which obey the Fermi statistics is practically independent of temperature except in the region of maximum energy. Fowler [6] and DuBridge [7] have applied Fermi statistics to investigate the temperature dependence of the photoelectric effect in this region (that is, near the threshold) and have found excellent agreement of the theory with experimental data. In view of these theoretical considerations very slight dependence of the photoelectric emission upon temperature would be expected for the temperature range of the present experiments. It is therefore difficult to explain the large decreases, here observed, of some 20 per cent. for temperature changes of only about 30° C. in terms of a pure temperature dependence of the photoelectric effect.

2. The curves obtained in the present work, after some 25 hours of good outgassing, in which the photoelectric current is plotted against the current through the metal are, like that of Martin, self-consistent and reproducible. Considered *separately*, the curves appear to have their explanation in a true temperature variation of the photoelectric emission. When, however, the entire set of curves obtained with a given specimen are considered as a *group*, inconsistencies and lack of reproducibility, from curve to curve, is exhibited. Thus, curves of the photoelectric current *vs.* current through the metal obtained at various stages throughout the outgassing, although consistent and reproducible within themselves, yield percentage decreases of different amounts varying from 8 to 25 per cent. Variations of this magnitude cannot be explained on the basis of a pure temperature variation of the photoelectric effect but have an obvious connection with the occluded gases known to be present. Furthermore, the

[6] R. H. Fowler, *Phys. Rev.*, **38**, 45 (1931).
[7] L. A. DuBridge, *Phys. Rev.*, **43**, 727 (1933).

photoelectric emission as a whole increases considerably throughout the outgassing which means that each successive curve is displaced farther up the photoelectric current axis. It likewise follows that reproducibility of the curve for decreasing currents with that for increasing currents cannot be used as evidence that the observed changes are true temperature variations of the photoelectric effect, for curves like II and III of Fig. 6, taken after a very slight amount of outgassing, are reproducible within two or three per cent. Also, in the preliminary work on bismuth, curves of the Shenstone effect were obtained which were reproducible within 2 per cent. in spite of the fact that it is practically impossible to outgas this metal.

It is therefore concluded that the decrease such as was observed in the "*a*" curve of Fig. 8 is not a pure temperature variation of the photoelectric effect but is due largely, if not wholly, to variation in the gas content of the metal. We may assume, as a satisfactory explanation of the phenomena, that a layer of electropositive gas is formed on the surface of the metal at room temperature. This electropositive layer is removed with increasing currents through the molybdenum and formed again with decreasing currents, the removal and formation being due to temperature changes. The removal of an electropositive adsorbed layer would increase the photoelectric surface work function of the metal and thereby cause a decrease in the photoelectric emission of the metal. This explanation is in harmony with the data obtained by Martin for, as observed by Hughes and DuBridge [8] and also by DuBridge and Roehr,[9] his value of the photoelectric work function of molybdenum is too low. This theory is also in agreement with the work of Langmuir [10] on the adsorption of gaseous layers on metals.

Despite all possible precautions there undoubtedly exist sources of supply for the formation of this electropositive gas layer under the conditions of these experiments. Due to the use of a thermocouple spot-welded to the rear face of the

[8] Hughes and DuBridge, "Photoelectric Phenomena," p. 96.

[9] L. A. DuBridge and W. W. Roehr, *Phys. Rev.*, **42**, 52 (1932).

[10] I. Langmuir, Iwadare Foundation Lecture, "Surface Chemistry" (1934); I. Langmuir, JOUR. FRANK. INST., **218**, 143 (1934).

molybdenum and essential for the temperature measurements, the outgassing temperature had as its upper limit about 1300° C. DuBridge and Roehr [9] have shown the necessity of outgassing molybdenum for many hundreds of hours at temperatures up to 2100° K., and the subsequent use of a "getter," before a check of the thermionic and photoelectric work functions could be obtained. Complete outgassing of this kind was impossible in the present series of experiments. Langmuir [10] states that a layer of oxygen on tungsten does not evaporate appreciably even when heated for many hours at 1500° K. in a high vacuum, although heating for as little as 20 seconds at 2007° K. is effective in evaporating the surface film. Another copious source of gas may be found in those parts of the apparatus that can be outgassed to a limited degree only. The entire glass part of the tube, together with those metal parts in contact with the glass, cannot be heated much above 500° C. These parts, undoubtedly, continuously emit gases even under the high vacuum conditions that were attained. No experiments to investigate the nature of the gases evolved by the glass and metal parts of the tube were conducted. The nature of the assumed electropositive layer is thus a matter of conjecture. However, the glass and metal parts of the tube were thoroughly washed with distilled water before evacuating so that water vapor certainly is present. There is also considerable evidence[11, 12] that a film of water vapor acts as an electropositive layer. It is quite reasonable, therefore, to consider the electropositive layer as a layer of water vapor or as a layer of hydrogen formed by the decomposition of water vapor. Langmuir [13] has shown that water vapor, coming off the glass walls of a vacuum tube, may be decomposed so as to form a layer of hydrogen on a tungsten filament. A layer of hydrogen, like water vapor, acts as an electropositive layer [11] in influencing photoelectric emission.

It is further to be observed that formation and removal of an electropositive layer would, for clean metals, be a reproduc-

[11] Hughes and DuBridge, "Photoelectric Phenomena," p. 78 (Table 3–3).

[12] H. Klumb, *Zeit. fur Phys.*, **47**, 652 (1928).

[13] I. Langmuir, Iwadare Foundation Lecture, "Fundamental Industrial Research" (1934).

ible process. The number of gas molecules striking the metal surface is given in statistical mechanics by the equation:[14]

$$Z = \frac{p}{(2\pi mkT)^{1/2}} \text{ per cm}^2 \text{ per sec.} \qquad (1)$$

Since the temperature in this equation refers to the temperature of the gas surrounding the metal, T may be considered constant for a change of 30° C. in the metal. The rate of evaporation of the gas from the metal surface, as given by the following formula,[10] is a function of the temperature of the metal:

$$n = Ae^{-w/kT} \text{ per cm}^2 \text{ per sec.,} \qquad (2)$$

where w is the heat of evaporation per atom and A is a constant independent of temperature. Thus Z is a constant for the temperature range in the present case whereas n is a variable. Since equilibrium between the rate of condensation and the rate of evaporation would exist at any given temperature of the metal, the number of adsorbed molecules, vacuum conditions remaining unchanged, would depend on the temperature of the metal and would vary as the temperature of the metal varied; thus being reproducible with temperature. All the curves of Fig. 8 are consistent with this explanation. In addition, the gradual elimination of gases *absorbed* in the body of the metal would cause the steady increase in photoelectric emission noted during the outgassing of the molybdenum.

To interpret the results given under "no outgassing" and "partial outgassing" requires a slight extension of the above electropositive adsorbed layer theory. As previously explained the initial large increases in photoelectric emission shown by curve 1 of Fig. 3, are undoubtedly of gaseous origin. The succeeding curves, in Fig. 3, show a gradual increase in sensitivity and are also characterized by the appearance of a maximum which is fairly reproducible. This behavior can readily be explained by assuming that the electropositive layer has a certain critical, possibly monatomic, thickness. After the formation of this critical electropositive

[14] R. C. Tolman, "Statistical Mechanics with Application to Physics and Chemistry," p. 206 (formula 437).

layer, the addition of more gas molecules to the metal surface has the effect of *reducing* the photoelectric emission of the metal. Thus, as the current through the metal is increased, the first effect is to remove these additional gaseous molecules and thereby to cause an *increased* photoelectric emission. Further increase of the current through the metal would then bring about the removal of the critical electropositive layer with the associated decrease in the photoelectric current. The photoelectric current would thus pass through a maximum as is observed. The curve for decreasing currents would reproduce this cycle provided there were sufficient molecules available to form both the critical electropositive layer and the additional "suppressing" layer. The curves under "no outgassing" were obtained with the first photoelectric cell containing two waxed joints which limited greatly the amount of possible outgassing of the glass parts of the cell. It is thus natural to suppose that sufficient gas molecules were present to produce the effect according to the hypothesis presented. There must also be considered the fact that the metal itself has been subjected to no outgassing. In view of this it is quite possible that the additional "suppressing" layer is formed by the diffusion of *absorbed* gases toward the surface. The gradual disappearance of the maximum on the curves, with subsequent outgassing of the molybdenum, supports this view. Hallwachs,[15] has suggested that the absorbed and adsorbed gases play different roles in relation to emitted photoelectrons. Our explanation is consistent with such a view.

The results given under the heading "partial outgassing" are to be interpreted much the same as those listed under "no outgassing." The curves of this group (Figs. 5*a*, 5*b*, 5*c* and 6) are marked by a further decrease in the maximum which, however, still appears at about the same value of the current through the specimen. The explanation of these curves according to the assumptions made is that definite outgassing, with the accompanying additional cleaning up of the molybdenum, has reduced the number of available gas molecules. This makes more difficult the formation of the

[15] Hughes and DuBridge, "Photoelectric Phenomena," p. 80.

additional "suppressing" layer. The curves therefore begin to show more marked decreases of the photoelectric sensitivity with increasing current. The gradual removal of occluded gases also has the tendency to make each stage in the process of degassing more stable. Thus the curves for increasing currents and for decreasing currents in Fig. 5*c* appear in the same relation to each other (that for decreasing currents below that for increasing currents) as do those of Figs. 5*a* and 5*b* although the order of taking the points in 5*c* is reversed from the order used in 5*a* and 5*b*. The curves of Fig. 6 also indicate that a rather stable state has been reached.

CONCLUSION. ACKNOWLEDGMENT.

The following points form a condensed summary of the results of this series of investigations: (1) The initial large increases in the photoelectric emission of bismuth with the passage of currents through the metal as reported by Shenstone and checked by the present author in a series of preliminary experiments, and also similar large initial increases in the case of molybdenum reported here, must be definitely attributed to the removal of occluded gases from the metals. (2) The observed variation of the photoelectric emission of molybdenum as a function of the current through the metal is not due to some obscure effect of the current itself but is connected with a variation of gas content of the metal superinduced by temperature change. These points are established by definite experimental evidence. A theory, based on surface chemistry, in which the variation of the photoelectric sensitivity of the outgassed molybdenum with the passage of a current through the metal is attributed to the formation and evaporation of an adsorbed electropositive layer of gas, is developed. This theory coördinates the large amount of data and offers a satisfactory general explanation of the observed effects.

In conclusion it is a pleasure to thank Dr. C. B. Bazzoni, Professor of Experimental Physics, who suggested this problem and aided materially in its completion by many helpful suggestions.

ABSTRACT.

This paper is a report on the "Shenstone effect" (the variation of the photoelectric sensitivity of a metal as a function of electric currents passed through it).

A systematic study of the effect of occiuded gases on the total photoelectric emission of molybdenum was made as a continuation of previous work of a similar character on bismuth. The experiments are divided into three groups corresponding to three stages of outgassing: (1) no outgassing; (2) partial outgassing; (3) good outgassing. Two photoelectric cells were used; one entirely free of waxed joints. The effect of temperature change on the "Shenstone effect" was also studied.

It is concluded: (1) that the initial large increases in the photoelectric emission of bismuth and molybdenum with the passage of currents through the metal (the original Shenstone effect) must be definitely attributed to the removal of occluded gases from the metals; (2) that the observed variations in the photoelectric sensitivity of molybdenum as a function of the current through the metal are not due to some obscure effect of the current itself but are connected with a variation of the gas content of the metal superinduced by temperature change. The observed variations in the photoelectric sensitivity of the outgassed molybdenum are attributed to the formation and evaporation of an adsorbed electropositive gas layer. This theory is shown to coördinate the large amount of data obtained.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

NINTH SESSION OF THE INTERNATIONAL COMMISSION FOR UNIFORM METHODS OF SUGAR ANALYSIS.

The ninth session of the International Commission for Uniform Methods of Sugar Analysis was held at the Institute for Chemistry, London, August 31 to September 5 under the presidency of Frederick Bates of this Bureau. Over 100 delegates, representing nearly all of the sugar-producing countries of the world, were in attendance, including representatives from Australia, Mauritius, and Hawaii. The program comprised the following 16 subjects:

1. Constitution and by-laws.
2. Weighing, sampling, and classification of sugars.
3. Conductometric determination of the ash content of raw sugars.
4. Determination of reducing sugars.
5. Determination of the decolorizing power and filtering quality of chars.
6. Testing of molasses.
7. Application of refractometric methods to sugar analysis.
8. The 100° point of the saccharimeter.
9. Standardization of quartz control plates.
10. Tests for the evaluation of the refining qualities of raw sugars.
11. Elimination of errors due to lead clarification.
12. Determination of raffinose.
13. Colorimetry in the sugar industry.
14. Values of Clergent divisors.
15. Determination of moisture.
16. Determination of the hydrogen-ion concentration.

In addition to the president of the commission, two other members of the Bureau's polarimetry section attended the session: J. F. Brewster and C. F. Snyder. The sessions were

* Communicated by the Director.

conducted in English, French, and German, although 16 different languages were represented among the delegates. The Bureau's representatives acted in the capacities of either referee or associate referee on a number of the subjects in the preceding paragraph.

One of the most important features of the session was the Bureau's paper on the French Sugar Scale. This scale is used to a considerable extent in France, Belgium, and certain colonial possessions. There have long been two groups among French physicists who were deadlocked on what physical values should be used in the standardization of the French Sugar Scale. In the present paper, the Bureau was able to demonstrate that the view of neither of these groups was the correct one. It now appears as though the National Bureau of Standards value for the French Sugar Scale will be adopted officially in France, thereby bringing the French scale into agreement with the International Sugar Scale. The latter is the National Bureau of Standards scale which was adopted by the International Commission for Uniform Methods of Sugar Analysis at Amsterdam in 1932.

The Tenth Session of the Commission will be held in Berlin in 1940.

TWELFTH CONFERENCE OF THE UNION INTERNATIONALE DE CHIMIE.

At the request of the National Academy of Sciences and the National Research Council, Frederick Bates attended the twelfth conference of the Union Internationale de Chimie as a delegate of these organizations. This session was held at Lucerne and lasted throughout the week of August 16. The American delegation was headed by Professor Bartow, president of the American Chemical Society, and consisted of six members. The Council of the Union Internationale de Chimie proposed to alter the Constitution of the Union in order that the International Commission for Uniform Methods of Sugar Analysis might become an integral part of the Union and still have complete autonomy in promulgating its findings. The Commission unanimously rejected the proposition that it become affiliated with the Union Internationale de Chimie.

However, there are other scientific organizations which may take advantage of the invitation of the Union Internationale de Chimie to affiliate with it.

CORROSION TESTS OF WELDED LOW-CARBON STAINLESS STEEL.

Laboratory tests are frequently used to determine the resistance of a material to corrosion under certain conditions. The value of these tests as an indication of the future behavior of materials in service depends largely upon the type of service and upon the interpretation of the results.

Several methods have been proposed for testing stainless steels, some of which are embodied in specifications for these materials. Three methods have been investigated recently by George A. Ellinger and Leon C. Bibber of the Bureau's metallurgical division, in connection with the welding of stainless steel containing about 0.06 per cent. of carbon.

A solution of copper sulphate in sulphuric acid did not noticeably attack either plate or weld metal that had been heated to any temperature, although this reagent has been reported to cause intergranular corrosion in stainless steels of higher carbon content.

Concentrated nitric acid was found to attack carbides which had been localized at grain boundaries. The acid attack destroyed the cohesion between the grains by dissolving or decomposing the carbides, thus allowing the grains to become loosened and ultimately to fall away from the material. This type of attack was found only in plate metal which had been heated to 650° C. and which contained carbides localized at the grain boundaries. Carbides in weld metal were usually found in areas of delta ferrite and not in the grain boundaries, regardless of heat treatment; consequently the attack by nitric acid on weld metal was slight.

A solution of hydrochloric acid and water (equal parts) attacked the plate metal severely. Heat treatment of the plate did not appreciably change this rate of attack. Weld metal that was not heat treated was attacked at approximately the same rate as plate metal. After annealing at either 650° C. or 980° C., weld metal was more resistant to attack than plate metal.

The complete report on this investigation will be published as RP963 in the *Journal of Research* for January.

INFRARED RADIATION FROM OTTO CYCLE ENGINE EXPLOSIONS.

The paper by Sidney Steel, Armistead Wharton, and C. H. Roeder, reviewed under the above title on page 121 of Technical News Bulletin 224 (December 1935), appeared in the January 31, 1936 issue of *Engineering* (London, England) and was followed in a later issue by an analysis and discussion of the data, completed by the senior author after leaving the Bureau. The complete paper has recently been issued as Publication No. 188 from the Graduate School of Engineering of Harvard University. (A limited number of copies are available. Applications should be made either to the Graduate School or to the senior author in care of the National Bureau of Standards.)

METHODS OF DETERMINING GLOSS.

For those who are interested in gloss as it affects the appearance of different materials, a paper in the January number of the *Journal of Research* (RP958) by Richard S. Hunter will be of interest because it brings together much of the available information upon gloss and discusses certain phases of the subject which have not heretofore received attention.

Glossiness is a characteristic common to many materials encountered in everyday life and is caused by surface reflection. Because surfaces are of so many types structurally, many kinds of glossy appearance are possible. The fundamental physical basis for describing the gloss of surfaces is provided by functions which describe the directional distributions of light reflected by these surfaces. However, such distribution functions are quite complex and, although they supply much more information about the reflectance properties of surfaces than do gloss determinations which give individual, arbitrary gloss values, they do not provide a practical means for the routine examination of the gloss of products in industry and commerce. What are desired for

"practical" measurements of gloss are instruments which give a single value of gloss for each surface measured.

If one studies the different types of glossy appearance, or if one examines the different instruments which are used to measure gloss, he discovers that there are several types of gloss. Six types of gloss have so far been described. They are: (1) specular gloss (shininess), (2) sheen (shininess at grazing angles), (3) contrast gloss (contrasts between high-light areas and non-high-light areas), (4) absence-of-bloom gloss (bloom is the reflection haze adjacent to reflected high-lights), (5) distinctness-of-reflected-image gloss, and (6) surface smoothness (absence-of-surface texture). Materials which are graded for gloss are usually evaluated in terms of one or another of these types of gloss. When one grades a series of samples for gloss, he unconsciously searches out and uses the appearance characteristic in which the differences are most noticeable. The six types of gloss identify six of these appearance characteristics.

The applicability of any instrument to a given gloss problem is best determined by actual trial in which comparisons are made between visual estimates of gloss and results obtained with the trial instrument on samples typical of the materials to be measured. Specular gloss measurements are most useful in the medium-gloss range, and measurements of distinctness-of-reflected-image gloss on samples in the high-gloss range. Instruments have not as yet been developed to measure bloom. Surface texture is best recorded by actual photographs of surfaces in specularly reflected light.

In the paper a number of gloss instruments are described and a bibliography is included which lists most of the previous publications on the subject.

TEXTILE TESTING IN RELATION TO CHEMICAL PROCESSING AND USE.

The following is an abstract of an address by William D. Appel, chief of the Bureau's textile section, before the Toronto Chemical Association at Toronto, Canada, on December 7, 1936.

Textile testing has undergone a marked change in recent years, largely as a result of the applications of chemistry and chemical engineering in the textile industry. Since the World War, several types of rayons, which differ in form, appearance, and properties, have come into general use. Mixtures of two and sometimes more of the major fibers have become common. New dyes, giving a wider range of colors, dyeing properties, and color fastness have been developed. New softening, stiffening, weighting, coating, proofing, and other agents and processes have been applied to fibers, yarns, and fabrics, in order to modify their properties and make the finished products more attractive and more useful. Further, the uses for textiles have become more varied, more specialized, and more exacting in their requirements. These developments have made it difficult if not impossible to judge the serviceability of textiles adequately from their constructions and compositions. Users are no longer concerned so much with how and of what textiles are made as they are with what textiles will do. There has arisen a need for means to test the performance or serviceability of textiles.

Some of the problems of textile testing, which have been studied at the Bureau from the point of view of the user of textiles, include: How to determine whether a colored material has adequate fastness to light; whether a silk, particularly a weighted silk, will deteriorate rapidly or will give reasonable service; whether a wool textile is exceptionally susceptible to deteriorative influences; the evaluation of resiliency, ability to drape, and resistance to creasing and crushing of a fabric.

Machines have been developed at the Bureau for testing the performance of carpets and rugs, hosiery, coated fabrics, window-sash cord, and shoes. Some of the more interesting results obtained with these machines are as follows: In accelerated tests, a window-sash cord containing wax lasted nearly ten times as long as the same cord without wax; rugs wore from $1\frac{1}{2}$ to $2\frac{1}{2}$ times as long when placed over a resilient underlay as when the underlay was not used; the index of wear of rugs given by the machine correlated well with the density of the pile, but if the wool in the pile had been overbleached the rug wore out rapidly; the upper part of the leg of

some silk stockings could be distended several thousand times, in a way simulating use, without developing holes or runs or bagginess, but other stockings failed in these respects when distended less than 100 times; some of the finishes applied to silk stockings to improve their appearance had a deleterious effect upon their performance.

Textile chemists and users of textiles have an interest in common in the performance type of testing. Both are concerned with the three factors which govern the suitability of textiles for particular uses: *Cost, serviceability,* and *appeal.* The chemist is constantly striving to lower costs, to increase serviceability, and to create new appeal in such characteristics of textiles as appearance, color, feel, and style, by chemical means. Laboratory performance tests can serve him as a guide in this work. They can be used to control processes and standardize products. When used in these ways, performance tests serve the user of textiles indirectly. They will be of the greatest help to users of textiles when they are incorporated in nationally accepted standards for the certification to the user of the serviceability of the textiles he buys.

COMMERCIAL STANDARD FOR MIRRORS.

The Bureau has just released a second edition of its Commercial Standard CS27–36, for mirrors. The first edition of this commercial standard, entitled Plate Glass Mirrors, was published in 1930. The revised standard covers plate glass mirrors and "shock" or common window glass mirrors. Plate glass mirrors are of three distinct grades designated as "A," No. 1, and No. 2 quality, based on the presence of certain defects which generally follow existing grades of plate glass. Shock or common window glass mirrors are manufactured in one quality only, based upon defects found in window glass. They are available in single strength, double strength, and heavy window glass. The standard covers thicknesses, silvering, quality designation, and guarantee, and method of inspection. A glossary of terms is also provided for those unfamiliar with the trade terminology sometimes employed.

Experience of the industry under the provisions of the

first edition of this standard indicated the desirability of eliminating AA and No. 3 quality plate glass mirrors.

The first edition covered plate glass mirrors only, while the second edition was extended to include "shock" or common window glass mirrors.

The personnel of the standing committee is given in the printed pamphlet, together with a list of acceptors, and a brief history of the project.

The revised standard became effective for new production on August 20, 1936. Copies are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

THE FRANKLIN INSTITUTE

ANNUAL MEETING, JANUARY 20, 1937.

The annual meeting of The Franklin Institute was held on the evening of Wednesday, January 20, 1937, at 8:30 o'clock, with Mr. Nathan Hayward, President, in the chair.

The first order of business was the reading of the report of the tellers of the annual election which indicated that the following gentlemen were duly elected:

President (To serve one year)	Philip C. Staples
Vice-Presidents (To serve one year)	Henry Howson
	Walton Forstall
	W. Chattin Wetherill
	S. S. Fels
Treasurer (To serve one year)	Benjamin Franklin
Managers (To serve three years)	Francis T. Chambers
	Irenee DuPont
	W. H. Fulweiler
	Nathan Hayward
	Horace P. Liversidge
	Malcolm Lloyd, Jr.
	Marshall S. Morgan
	Coleman Sellers, 3rd
(To serve one year)	Clarence L. Jordan
	Conrad N. Lauer

The report was adopted and the Chairman declared the gentlemen named elected to the several offices. The thanks of the meeting were extended to the tellers for their labors. The Chairman expressed regret at the absence of the President-elect, which was due to illness.

It was announced that the minutes of the December meeting had been printed in full in the January JOURNAL and as there were no corrections or additions they were approved as printed.

The Secretary reported that 34 members had been added to the membership of the Institute and 90 to the Museum membership, making the total of all classes of membership at this time 6,005.

The Secretary then made the following remarks: "The most important recent event, I think, has been the resignation of our good friend and President, Mr. Hayward. Fortunately, Mr. Hayward will remain on our Board of Managers and will participate actively in the affairs of the Institute. He has merely asked to be relieved of the arduous duties which devolve upon the incumbent of the office of President, which duties he has carried on so well during the past eight years. Due to his inherent kindliness, his tact, and his keen business sense, The Franklin Institute has gone forward, under his guidance, extending always its century-old reputation."

Mr. Albert F. Murray, engineer in charge of television of the Philco Radio and Television Corporation, was then introduced and spoke on the subject: "Television Progresses." After outlining the beginnings of television, the speaker described the advances made during recent years. With the aid of lantern slides and apparatus, he described the functioning of the present system with its special vacuum tubes. The experiments and field tests carried on during the year 1936 included broadcasting, home reception and television standards. Unsolved problems and future accomplishments were also discussed.

At the close of Mr. Murray's paper, the retiring president, who had left the meeting, made a brief address from a television broadcasting station several miles from the lecture hall. He was visible to the audience and his remarks were received with great applause. Adjourned.

HENRY B. ALLEN,
Secretary.

(Further demonstrations were made in the hall adjoining the lecture hall after the meeting.)

REPORT OF THE SECRETARY AND DIRECTOR OF THE FRANKLIN INSTITUTE.

In accordance with our usual custom, I should like to make a brief report of the activities of The Franklin Institute during the past year to its members.

An important departure during the past year was the establishment of a department for membership and fund raising. Dr. Sidney L. Wright has been appointed to study the whole question of membership, its various classifications, its needs, and its relationship to the Institute as a whole. During the past year the membership in all branches has been increased by 2,781, making now a total of 6,005.

It is the sincere desire of the governing body of the Institute to make this building of the greatest possible service to the members of The Franklin Institute.

I wish to take this opportunity also to acquaint you with a new committee—The Hostess Committee of The Franklin Institute. I wish to express publicly our appreciation to this group of ladies for their splendid coöperation in the affairs of the Institute. They meet monthly and by their interest have widened the scope of our activities and have made the Institute better known among our people. They are always on call to aid in the entertainment of distinguished visitors and have given most generously of their time and means. We are hopeful that the work of this important group will long continue.

The Museum and the Planetarium continue to prove of paramount importance both in the field of education and the field of amusement. We are glad to report a steady increase in the number attending the Museum and the Planetarium demonstrations. As you probably know, admission is free to the school children of the City of Philadelphia. To those outside the city an educational discount is granted, with the result that children from the surrounding towns and cities visit The Franklin Institute in large numbers. During the month of December for the year 1936, we had the largest number of people for any one month since the Planetarium was opened. Approximately 34,000 enjoyed our presentation of "Skies of the First Christmas."

The JOURNAL OF THE FRANKLIN INSTITUTE has been the medium of introducing to the scientific world many papers of importance, among them one by Dr. Einstein.

The Library's usefulness has increased to such an extent that it has been necessary to open it two nights a week.

The lectures continue to bring to the Institute hundreds of members and many of their friends. They have proven so popular with the membership that it has become necessary to reserve seats for those who carry membership cards. The Tenth Anniversary of the Christmas Week Lectures for Young People was celebrated during the past year, with the largest attendance on record for this special series.

These are the routine activities of The Franklin Institute. I wish to call them to your attention at this time in order that you may know they are not neglected during our efforts to bring you and the public to a more intimate knowledge of our great technical and industrial museum.

We have three other major activities which I wish to mention:

Medal Day, the third Wednesday in May, when all the medals recommended by the Committee on Science and the Arts are awarded. Last year you will remember two outstanding Americans were honored—Dr. Jewett of the Bell Telephone Company and Dr. Kettering of General Motors Company. The work of this Committee is now carried on under the able guidance of Dr. John Frazer, who was appointed last June to fill the vacancy caused by the death of Dr. G. A. Hoadley, Secretary of the Committee on Science and the Arts, who was known and loved by many of you.

The Bartol Research Foundation of The Franklin Institute continued to blazon a bright path across the field of science, bringing added honors to this old Society. A complete report on the work of these laboratories will be given at our April meeting by Dr. W. F. G. Swann, Director of the Foundation.

Our newest branch of endeavor, The Biochemical Foundation of The Franklin Institute, is flourishing under its director, Dr. Ellice McDonald. Dr. McDonald gave a full report on the work of his laboratories at the December meeting of the Institute, so that I need not reiterate it at this time.

I hope this brief résumé will interest you. The name of The Franklin Institute is broadcast to the world through many mediums. It is always our endeavor to make this old society as widely useful as possible. It is our hope that, with the able assistance of our new President, and the continuing coöperation of our retiring President, you may this year have more cause than ever to be proud that you are a member of The Franklin Institute.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, January 13, 1937.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JANUARY 13, 1937.

MR. JOHN A. VOGLESON *in the Chair.*

The following reports were presented for final action:

No. 3018: Proving Ring.

This report recommended the award of an Edward Longstreth Medal to Mr. Herbert L. Whittemore, of Washington, D. C., "In consideration of his part in the invention and development of the Proving Ring for the direct checking of calibrations of testing machines."

No. 3026 / No. 3027 } The Standing Committee on the Franklin Medal presented its report for final action making the following recommendations:

"That the recipients of the two Franklin Medals to be awarded in 1937 be:

"DR. ROBERT ANDREWS MILLIKAN, of Pasadena, California, 'In recognition of his isolation and measurement of the fundamental unit of electricity, the electron; the photoelectric determination of the fundamental constant of radiation, Planck's constant; the extension of the ultraviolet spectrum by two octaves to join the spectrum of soft X-rays; and the study of the nature and the properties of a very penetrating radiation of cosmic origin';

"And DR. PETER JOSEPH WILHELM DEBYE, of Berlin, Germany, 'In recognition of his fundamental conception and masterly development of the theory that the molecules of many substances possess permanent dipole moments,—a theory of great value in stimulating a vast amount of fruitful research on the electrical properties of insulators; his extension and generalization of Einstein's theory of the specific heats of solids; and his work, in collaboration with Hückel, on a theory of the thermodynamic properties of electrolytic solutions,—work upon which the whole modern theory of electrolytes is based.'"

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

RECENT ADDITIONS.

Agriculture.

Association of Official Agricultural Chemists, Official and Tentative Methods of Analysis. Fourth Edition, 1935. 1936.

Architecture and Building.

American Concrete Institute. Proceedings. Volume 32. 1936.

BRETT, THOMAS J. Air Conditioning-Design and Construction of Ducts. 1936.

CHENEY, SHELDON, AND MARTHA CANDLER CHENEY. Art and the Machine. An Account of Industrial Design in 20th-Century America. 1936.

Astronomy.

BARNS, CHARLES EDWARD. 1001 Celestial Wonders. Third Edition. 1931.

PROCTOR, MARY. Romance of the Moon. 1928.

Biography.

BRODETSKY, S. Sir Isaac Newton. A Brief Account of his Life and Work. Second Edition. 1929.

HART, IVOR B. The Great Engineers. 1928.

HOLMYARD, ERIC JOHN. The Great Chemists. Third Edition. 1929.

TURNBULL, H. W. The Great Mathematicians. Second Edition, Revised. 1933.

Chemistry and Chemical Technology.

Beilsteins Handbuch der organischen Chemie. Vierte Auflage. Fünfundzwanzigster Band. 1936.

FINDLAY, ALEXANDER. The Teaching of Chemistry in the Universities of Aberdeen. 1935.

GARDNER, WILLIAM. Chemical Synonyms and Trade Names. A Dictionary and Commercial Handbook. Fourth Edition, Much Enlarged. 1936.

GLASSTONE, SAMUEL. Chemistry in Daily Life. Second Edition. 1929.

GRIFFITH, R. H. The Mechanism of Contact Catalysis. 1936.

SCHMIDT, JULIUS. A Text-Book of Organic Chemistry. English Edition by H. Gordon Rule. Third Edition, Revised and Extended. 1936.

THOMPSON, R. CAMPBELL. A Dictionary of Assyrian Chemistry and Geology. 1936.

Electricity and Electrical Engineering.

DUDLEY, A. M. Connecting Induction Motors. Third Edition. 1936.

ESPY, W., AND M. KNOLL. Werkstoffkunde der Hochvakuumtechnik. 1936.

MOON, PARRY. The Scientific Basis of Illuminating Engineering. First Edition. 1936.

Engineering.

American Society for Testing Materials. A. S. T. M. Book of Standards 1936 Parts 1–2. 1936.

RATZERSDORFER, JULIUS. Die Knickfestigkeit von Stäben und Stabwerken. 1936.

Geology.

DACHLER, ROBERT. Grundwasserströmung. 1936.

Hygiene and Physiology.

CAREY, EBEN J. Medical Science Exhibits. A Century of Progress. Chicago World's Fair 1933 and 1934. 1936.

GOODMAN, HERMAN. Cosmetic Dermatology. First Edition. 1936.

Mathematics.

DAVIS, HAROLD T. The Theory of Linear Operators from the Standpoint of Differential Equations of Infinite Order. 1936.

EUKLID. Die Elemente. Nach Heibergs Text aus dem Griechischen überstezt und herausgegeben von Clemens Thaer. iv Teil (Buch x). 1936.

Mechanical Engineering.

ALTHOUSE, ANDREW D., AND CARL H. TURNQUIST. Modern Electric and Gas Refrigeration. 1936.

JONES, FRANKLIN D. Ingenious Mechanisms for Designers and Inventors. Volume 2. First Edition. 1936.

KATES, EDGAR J. Diesel Electric Plants. 1936.

Metallurgy.

American Institute of Mining and Metallurgical Engineers. Transactions. Iron and Steel Division. 1936.

WOLDMAN, NORMAN E., AND ALBERT J. DORNBLATT. Engineering Alloys: Names, Properties, Uses. 1936.

Naval Architecture and Navigation.

DARLING, THOMAS. Miniature Racing Yachts and How to Build Them. 1936.
NANCE, R. MORTON. Sailing-Ship Models. A Selection from European and American Collections with Introductory Text. 1924.

Photography.

Deutsche Lichtbild. Jahreschau 1937.
Photograms of the year 1937.

Physics.

EYRING, CARL F. A Survey Course in Physics. 1936.

Printing.

FRIEND, LEON, AND JOSEPH HEFTER. Graphic Design. 1936.

Sugar.

TROMP, L. A. Machinery and Equipment of the Cane Sugar Factory. 1936.

BOOK REVIEWS.

WERKSTOFFKUNDE DER HOCHVAKUUMTECHNIK, EIGENSCHAFTEN, VERARBEITUNG UND VERWENDUNGSTECHNIK DER WERKSTOFFE FÜR HOCHVAKUUMRÖHREN UND GASGEFÜLLTE ENTLADUNGSGEFÄSSE, von W. Espe und M. Knoll. 383 pages, tables, illustrations, 16.5 × 25 cms. Berlin, Julius Springer, 1936, price 48 marks.

This book is aimed to satisfy the need for a combined presentation on the quality, the manufacture and the application methods of the materials used in vacuum and gas filled tubes. The authors, in the preface, state that the occasion for this work has been the small differences and gaps in knowledge that teaching experience has opened up, creating an unsatisfied feeling. Unfavorable opinion on the success of a series of tubes often is taken from the poor performance of a single tube and this danger can only be avoided by a clear understanding of physical principles. Because this may retard industrial and scientific work with vacuum and gas filled tubes the authors here attempt to clear up the matter by presenting factural data for direct application.

The book is written with the knowledge that differences of opinion exist regarding the desirability of sharp limitations for each chapter or whether the different branches of vacuum tube technic are interwoven. With this as a background the style of presentation is most helpful. High melting metals, platinum, base metals and their alloys, quicksilver, earth metals, graphite, glasses, quartz, ceramics, luminous material, gases and vapors are each taken up separately and given thorough treatment from the production through combinations, chemical and physical properties to the technic of utilization in the particular part of the tube. There are also included the fundamentals of metallography from the utilization point of view. Since high temperatures are involved with tube electrodes in their operation and manufacture, there are considered here the thermal constants of the materials as well as crystal structure and chemical reactions at high temperature.

Special attention is given to materials whose applications may influence other materials. There are treated in detail hot and cold cathodes, photoelectric cathodes and gas absorption as well as relationships of different materials under particular conditions encountered.

At the end of the book there is quite a complete bibliography of reference matter and a useful subject index. The book is well illustrated with drawings and photographs. The text contains a fund of valuable data well presented that should find ready usefulness.

R. H. OPPERMANN.

THERMODYNAMIC PROPERTIES OF STEAM, INCLUDING DATA FOR THE LIQUID AND SOLID PHASES, by Joseph H. Keenan and Frederick G. Keyes. 89 pages, tables, diagrams, 19.5 × 26 cms. New York, John Wiley & Sons, Inc. London: Chapman & Hall, Ltd., 1936. Price $2.75.

This is a book of eleven tables representing the latest information available. They include two tables on saturation—one on temperatures and the other on pressures, a table on superheated vapor, compressed liquid, saturation—solid vapor, viscosity, heat conductivity, conversion factors, thermometer calibration factors, logarithms to the base 10 and to the base e. Certain charts of the specific heat hitherto not available are included which show the extraordinarily rapid changes and the large values of the quantity corresponding to volumes at and near the critical state. The scales of the charts are selected to permit reasonable accurate readings of the variables.

More than six years have elapsed since the appearance of the Keenan Steam Tables and Mollier Diagram which were the result of considerable research. Now, the available data on water are considerably increased and with the total research covering a period of over a decade in the United States, Germany, England and Czechoslovakia it was deemed unnecessary to further delay in publishing revised tables. The present tables were computed from entirely new formulations of the properties of water. The results appear satisfactory, at least as far as can be judged by applying the usual criteria of thermodynamic consistency.

R. H. OPPERMANN.

LESSONS and PROBLEMS IN ELECTRICITY, by Newell C. Page. 356 pages, diagrams, 14 × 22 cms. New York, The Macmillan Company, 1936. Price $2.75.

This is an interesting text not only because it is on an interesting subject, the fundamentals of electrical engineering, but because of its simplicity of presentation. In this respect the book can be classed with the modern text, the class which has been developed through experience to the point of having the most sympathy with and facilitating study for the serious-minded follower of the subject. The book is meant for those whose prerequisites include mechanics and first year calculus.

The coverage is sufficient to provide entry to pure science, the physical being emphasized rather than the engineering features. Electrostatics, current and EMF, Ohms, Kirchoff's and Joule's laws, energy and power, thermo-electricity, magnetism and magnetic fields, capacitance and inductance, gaseous conduction and the electron, and conduction in high vacua are all given treatment. The latter two topics are perhaps best illustrative of the outstanding feature of the book—simplicity in presentation, a feature that it is well to consider before undertaking

study. The text then proceeds through alternating currents, power factor, and the transformer. In these, graphical representations are dealt with by the use of vectors, incidentally giving a good ground work for their use in electrical engineering. Oscillatory circuits, and electromagnetic radiation provide a rounding out of the subject of the book. The last chapter is devoted to electrical units and dimensions in which there is explained the electrostatic system and the electromagnetic system. The book certainly covers a large field in a remarkably clear manner. It is so written that the student does not necessarily have to follow in the order of presentation of the various topics. Some can even be left out.

At the end of each chapter there are wisely constructed problems that will enable an impression to be made of the practical knowledge of the text. Answers are given to each problem.

A subject index completes the book. Many electrical engineers, can find in this book helpful and informative matter on subjects with which they may not have had recent experience.

R. H. OPPERMANN.

CYANIDATION AND CONCENTRATION OF GOLD AND SILVER ORES, by John V. N. Dorr. 485 pages, 15.5 × 23.5 cms. New York and London, McGraw-Hill Book Company, 1936. Price $5.00.

This work is one of the most important contributions to mining literature which has appeared in recent years. Written by a man who has distinguished himself in a vitally important branch of metallurgy and milling, and who now gives to the public the technical results of many years in chemical research and mechanical invention. My old friend John, or I should perhaps say Doctor Dorr, tells us in the preface something of his early technical career. Educated as a chemist, forced by circumstances to supplement chemistry with mechanical devices in a small western mining camp, he achieved a long step in the successful application of the then new and undeveloped MacArthur-Forrest cyanide solution process for the treatment of gold and silver ores.

Now let us digress for a moment to consider the effect and tremendous stimulus this epochal invention of the application of alkaline cyanide solutions had upon the gold mining industry of forty or fifty years ago. It is no exaggeration to say its importance to this world-wide industry is comparable in the field of metallurgy only to that of the Bessemer process in the production of steel. It is true it did not give us cheap gold as the latter produced cheap steel rails which in turn enabled the subsequent enormous extension of railway construction in many parts of the world. But it did lead to a marked increase of gold production by making it economically possible to treat ores hitherto considered of little or no value at a time when the output of gold, especially in the United States, indicated a steady and continued decrease.

This decrease, before the advent of the modern gold-dredge, no doubt was chiefly due to the exhaustion of alluvial or placer deposits cheaply worked and soon gone. The milling practice for gold ores fifty years ago was crude with a consequent low recovery of the precious metals. Moreover, the world's supply of gold lobe ores which were rich enough to treat economically at that time was relatively small; except for the use of mercurial amalgamation, which can hardly be considered a chemical reaction, no chemistry was used. Ores averaging under

ten dollars (on the $20.67 basis) per ton, even when not complex, were usually not profitable. Thus many mines which today could be successfully worked were then abandoned, most of them permanently. Many ores containing gold and silver are complex combinations of minerals carrying other metals mostly in the form of sulphides and arsenides which require very difficult treatment by amalgamation, in fact even with the so-called free milling ores the extraction seldom approached the rate now usually experienced with cyanide treatment. The result was that forty or fifty years ago the world's gold production indicated evidences of falling behind the needs of steadily increasing commerce and industry.

Dorr states that in South Africa alone the cyanide process made possible an increase of the gold production of $6,000 in 1890 to $6,000,000 in 1893 (vide page 3). These figures seem almost incredible, but it is likely the author has checked them. The paradox of the present situation regarding our gold supply is that the greater the output the higher its price in dollars. This has been caused by governmental juggling, the reasons for which seem to me past finding out. Its advantages to the gold mining industry have, however, been enormous since gold at $35.00 per ounce has enabled the miner to exploit ores hitherto absolutely unprofitable, even by the adoption of the most improved mining and milling methods. It is evident therefore that the gold production of the world is likely to steadily increase for some years to come, despite the fact that all mineral deposits are wasting assets whose valuable contents can never be replaced or renewed. The public in general and bankers and government officials in particular, seem unable to appreciate this fact. In other words, the miner cannot grow crops of minerals, and the supply of metallic ores, mineral fuels, oil, gas, etc. is not unlimited, and modern civilization is exhausting our resources at a most extravagant rate.

Doctor Dorr sketches the history of the development of the cyanide process from its origin about the year 1887 to the present in Chapter I, which also includes tables giving the "Annual Output of the World's Leading Gold Mines, 1929–1935."

Chapter II covers the various phases of preliminary examination and testing as a prerequisite to the best and proper methods to be adopted for any particular character of ore and conditions implied.

Chapter III deals with coarse crushing and Chapters IV and V respectively with sorting and fine grinding. Chapter VI deals with classification, a field in which Dorr's own inventions have been conspicuously successful, as also with sand and lime treatment, both phases of which are covered in Chapters VII and VIII. Chapter IX discusses the various methods of ore concentration and Chapter X the usual methods of bullion recovery. Plant control in Chapter XI is essentially chemical and has to do with the solvent solutions. Chapter XII on the treatment of simple and complex ores covers 138 pages and is the longest in the book. The treatment of old residues, costs and power consumption and "Useful Reference Information" is included in the following three chapters. Altogether the book is replete with useful information for the mining engineer and especially for those particularly concerned with the milling and the metal extraction of gold and silver ores.

The subject of this notable book tempts one to venture some reflections upon

the use of the precious metals in our modern civilization—observations and thoughts philosophical perhaps rather than practical and feasible.

Just how necessary or essential is gold in our every-day economy? Could we not to a great extent get along very well without it? As a matter of fact, as far as money is concerned, is it any use whatever?

The answer seems to me obviously we can because gold does not necessarily enter into our daily business and common transactions; probably in the course of time its absence would not be missed or substantially regretted. The savage, the pre-historic man, even the present day esquimaux has no use for it other than an urge for ornamentation or the adornment of his person, for of all the metals gold is the most beautiful, and as such ultimately evolved into a token of exchange value among primitive people and later as civilization developed became together with silver the basis of money.

If a government can say by fiat to the people that your dollar henceforth will be worth, say, only sixty cents in gold value and by tacit agreement the people subsequently find the dollar buys as much or as little as it did before, what use or need has there been or is there even now for gold? This is especially the case when government simultaneously decrees it a crime to possess or use gold coin or even hold any considerable amount of gold bullion. The logical and inevitable consequence of such a law is that the government accumulates a vast store of the metal to be buried in great mausoleums especially constructed for that purpose on the perfectly untenable postulate that such a hoard becomes the foundation of national credit, the guarantee of currency. As a matter of fact it is nothing of the kind as long as people and a nation pay their debts with whatever currency or acceptable medium of exchange is in common use or demand; credit automatically is established and maintained as long as the demand continues.

Raising the value or price of gold per unit ounce from $20.67 to $35.00 was certainly a great stimulus to the gold mining industry the world over, especially as a few years ago the outlook for a continued or even steady, much less a greater increase in the output of gold was anything but favorable. Thus for instance, when I was in South Africa six years ago, the official estimate of the future life of the Witwatersrand mines in the Transvaal was only ten years, whereas now by reason of the enhanced price of gold which makes it profitable to mine ores hitherto too lean to pay, has extended this limit indefinitely. The same thing has happened all over the world as far as gold mining is concerned.

The whole subject reeks with dissonance. The public bought bonds payable in gold coin of a definite standard weight and fineness and are now being repaid but sixty per cent. of the original outlay. That is a fine thing for governments, especially one like the Russian which does not pay back anything, although Russia is now the second largest producer of gold in the world, something which, I believe, never happened before in its history. On the whole, the increase of the price of gold to $35.00 per ounce is a good thing and an inflation which seems to have done more good than harm. The morality of taking the public's dollars at one hundred cents and paying only sixty cents is quite a different matter.

F. LYNWOOD GARRISON.

INTRODUCTION TO THEORETICAL CHEMISTRY, by W. B. Meldrum and F. T. Gucker. 614 pages, illustrations, 8vo. New York, American Book Company, 1936. Price $3.50.

The title might well have been supplemented with: For college students in their second year of chemistry. Such a book as this represents the materialization of the hopes—or fears—probably entertained by chemistry students twenty years ago. Even in those days it did not require a crystal gazer to foresee that the knowledge related to chemistry might easily double within the next score of years. So today the second year chemistry student is being thoroughly grounded in certain fundamental elements that former students did not learn until they turned upper classmen or, more often, postgraduates.

The authors, both professors of chemistry, have been fully aware of the inadequacy of a first year college course in general chemistry to cover the field satisfactorily. For some years they have been giving supplementary lectures covering the recent developments in the chemistry field and finally had the courage and energy to embody the same in the form of the text here under discussion. Fortunately, the authors had to establish more or less their own precedents as they went along. They have not followed blindly the texts of earlier books and have made earnest attempts to free their work from all "erroneous doctrine."

To list each chapter title would make burdensome reading. However, the naming of a few will give a better idea of the book's comprehensive scope. In addition to the more or less classical topics such as the elements and their classification, chemical combination and atomic theory, the gas laws and the kinetic theory, the reader will find well developed expositions on modern theories of electrolytes, radiations and spectra, radioactivity, isotopes, atomic structure, transmutation of the elements and the electronic theory of valence.

Problems and exercises have been appended to most of the chapters. The use of summaries also is very commendable. This work steps out of the usual run of textbooks class in yet another respect: "References are given throughout. Some are in the form of foot-notes to indicate sources of data and to call attention to original papers on the topics mentioned. References at the end of the chapters provide a more extensive reading." The subject matter is well illustrated by means of graphs, line drawings and reproductions of original photographs. Portraits of well-known scientists in the field of chemistry furnish an added touch of interest.

T. K. CLEVELAND.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 562, Air Flow in the Boundary Layer Near a Plate, by Hugh L. Dryden. 26 pages, tables, diagrams, 23.5 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

The published data on the distribution of speed near a thin flat plate with sharp leading edge placed parallel to the flow (skin friction plate) are reviewed and the results of some additional measurements are described. These experiments were carried out at the National Bureau of Standards with the cooperation and financial assistance of the National Advisory Committee for Aeronautics for the purpose of studying the basic phenomena of boundary-layer flow under simple conditions.

When the distribution of mean speed is measured in a stream without pressure gradient, it is found that the flow for some distance from the leading edge corresponds to that derived theoretically by Blasius from Prandtl's equations for laminar flow. At a definite value of the Reynolds Number formed from the distance to the leading edge and the speed of the stream at a considerable distance from the plate, the flow departs from the Blasius distribution and after a long transition region has the characteristics of fully developed turbulent flow, hereinafter designated "eddying flow." The Reynolds Number at which transition occurs is a function of the initial turbulence of the air stream, decreasing as the turbulence is increased.

Small pressure gradients in the air stream greatly change the critical Reynolds Number for a given turbulence.

From measurements of the amplitude of the u-fluctuation of speed it was found that the laminar region exhibits comparatively large fluctuations induced by the turbulence of the general flow. The laminar and eddying regions cannot be distinguished on the basis of the magnitude of the speed fluctuation, but the principal fluctuations in the eddying region are of higher frequency than those occurring in the laminar region.

Report No. 568, The Quiescent-Chamber Type Compression-Ignition Engine, by H. H. Foster. 20 pages, illustrations, tables, 23.5 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

The performance of a single-cylinder 4-stroke-cycle compression-ignition engine having a vertical disk form of combustion chamber without air flow has been determined. The number, size, and direction of the orifices of the fuel-injection nozzles used were independently varied. A table and graphs are presented showing the performance of the engine with different nozzles; results of tests at different compression ratios, boost pressures, and coolant temperatures are also included.

The best unboosted performance was obtained at a compression ratio of 15.3 at an engine speed of 1,500 revolutions per minute, using water as a coolant. The increase in indicated mean effective pressure with boost pressure was proportional to the increase in weight of inducted air for equal air-fuel ratios and comparable maximum cylinder pressures. The engine operation was smoother with boosting.

The engine power and fuel economy obtained with a 6-orifice nozzle was equal to or better than that obtained with nozzles having any other form, number, or combination of orifices. The optimum value for the number, direction, or size of the orifices is not sharply defined. Results indicate that impingement of the fuel spray on the piston and chamber walls, although it may be detrimental to efficient combustion, may aid distribution and consequently increase the power output. Although the results do not afford a rational basis for nozzle design that can be reduced to an analytical or empirical formula, they do show that engine performance can be improved by careful design of the injection nozzle.

The large percentage of the total fuel in the relatively solid spray core injected from round-hole orifices and the short time available preclude the probability of obtaining a homogeneous mixture of the fuel and air in a quiescent combustion chamber using a multiple-orifice nozzle. The resultant inferior performance compared with that obtained from the same combustion chamber with forced air

flow, despite the easy starting, easy scavenging, low mechanical losses, and freedom from knock, renders the quiescent-chamber engine unattractive for aircraft-engine use.

Report No. 571, Pressure Distribution Over a Rectangular Airfoil with a Partial-Span Split Flap, by Carl J. Wenzinger and Thomas A. Harris. 12 pages, diagrams, 23.5 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

Pressure-distribution tests of a Clark Y wing model with a partial-span split flap were made to determine the distribution of air loads over both the wing and the flap. The model was used in conjunction with a reflection plane in the N.A.C.A. 7- by 10-foot wind tunnel. The 20-per cent.-chord split flap extended over the inboard 60 per cent. of the semi-span. The tests were made at various flap deflections up to 45° and covered a range of angles of attack from zero lift to approximately maximum lift for each flap deflection.

The results are given as airfoil section, or rib, pressure diagrams for the wing with flap neutral. Increments of airfoil section pressures are given for various amounts of flap deflection so that combined wing and flap section pressure diagrams may be obtained by a simple addition. Calculated coefficients of section loads and moments and of wing loads and moments are also given for the wing and flap combination and for the flap alone.

It was found that deflecting a partial-span split flap affected the pressures and the section normal-force and pitching-moment coefficients over the entire wing span. The flap loads and moments were almost constant over the span of the partial-span flap for a given angle of attack and flap deflection. The maximum normal-force and hinge-moment coefficients were about the same for the partial-span split flap of the present tests as for a full-span split flap previously tested.

PUBLICATIONS RECEIVED.

Mechanics, Molecular Physics, Heat and Sound, by Robert Andrews Millikan, Duane Roller and Earnest Charles Watson. 498 pages, illustrations, 16 × 24 cms. Boston, Ginn and Company, 1937. Price $4.00.

Senior Science, Socialized for the High School, by George L. Bush, Theodore W. Ptacek and John Kovats, Jr. 835 pages, illustrations, 16 × 24 cms. New York, American Book Company, 1937. Price $2.20.

Properties of Matter, by F. C. Champion and N. Davy. 296 pages, illustrations, 15 × 23 cms. New York, Prentice-Hall, Inc. 1937. Price $4.50.

Quantitative Organic Microanalysis of Fritz Pregl, by Dr. Hubert Roth, third English edition, translated from the fourth revised and enlarged German edition, by E. Beryl Daw. 271 pages, illustrations, 15.5 × 25.5 cms. Philadelphia, P. Blakiston's Son & Co., Inc. 1937. Price $5.00.

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CURRENT TOPICS.

Cotton Drier Wins Favor of Growers and Ginners.—(U. S. Dept. of Agric. Clip Sheet.) Cotton from gins equipped with the new hot-air-blast drier developed in 1930 by engineers of the United States Department of Agriculture is bringing premiums consistently. As a result manufacturers of the equipment have been unable to keep up with the demand this year. Reports from the Delta region of the Cotton Belt and other humid sections of the belt indicate that gins without the new equipment are losing customers to the drying gins. According to estimates there are now about 400 of the new-type driers in the South. Twenty-five were installed in Louisiana alone this summer and observers estimate that even more will be put in next season.

The most important gain resulting from drying the seed cotton is in the improvement in grade which adds an estimated $2 to $7 a bale. In addition, the ginning process is speeded up and picking can be continued even in damp weather.

Satisfaction with the new equipment has been expressed by many growers and ginners. Farmers in one locality told a ginner that if he did not put in a drier he might just as well " built a fence around his gin." Reports are that many gins without driers closed down early in the season. One ginner tersely expressed the advantage of the drier when he said " now every day is a sunny day in this business." He also said, " Most of our troubles are over because the drying cuts down chokage of the gin, the machinery runs better, fewer employees are injured, and there is less danger of fire." Insurance companies have noticed these advantages and already there have been some reductions in rates.

C. A. BENNETT, of the Bureau of Agricultural Engineering Laboratory at Stoneville, Miss., inventor of the new drying equipment, which has been patented by the United States Department of Agriculture, says the widespread interest in this new development has brought many ginners and planters to the laboratory, where studies are being made of many aspects of the storing, handling, and processing of cotton from picking to marketing.

R.

Farm Fire Loss.—(U. S. Dept. of Agric. Clip Sheet.) Rural fires in recent years have taken annually an average toll of 3,500

lives and piled up a property loss estimated at $100,000,000, according to reports of the United States Department of Agriculture. However, noticeable reduction in losses have been brought about in communities where rural firemen have made a systematic study of fire hazards and conducted campaigns for their elimination.

R.

U. S. Army Establishes Precision Instrument Laboratory for Metropolitan Area at N. Y. U.—THORNDIKE SAVILLE, dean of the New York University College of Engineering, announced recently that the Ordnance Department of the U. S. Army would establish a precision gauge instrument laboratory for the metropolitan area early in 1937 in the college. The gauge laboratory, the only one of its kind in this area, will be used in the testing of various tools, instruments, mechanisms, and other equipment for military purposes. It will be available also to students of engineering, and others, for general research purposes. Instruments, capable of measurements to the millionth of an inch, will be certified by the U. S. Bureau of Standards before their installation.

R. H. O.

The Nickel Industry in 1936.—An annual survey prepared by ROBERT C. STANLEY, President, The International Nickel Company of Canada, Ltd. states that statistics for the first ten months of 1936 indicate that this year will establish new records for the industry in the volume and diversification of world consumption. This consumption of nickel in all forms attained a total of some 162,000,000 pounds, an increase of more than 20 per cent. over the corresponding period for 1935 and compares with 112,000,000 pounds in the first ten months of 1929. The steel industry, which is the dominant consumer of nickel, broadened its markets and in consequence substantially increased the use of nickel alloy steels. New compositions of nickel alloy steels in the form of sheets, plates and other mill products for structural purposes, not available heretofore, are now being marketed in considerable tonnages under a long list of trade names. Of interest have been the advances made in the stainless steel industry during the past year. Not only is more nickel being used, but the standards of quality of finished sheet and strip produced by the principal manufacturers are being steadily raised. In addition to these two conventional products in the stainless steel series of alloys there is a marked increase in output of wire and tubes, and the production of special alloy compositions suitable for machining has further expanded consumption. Con-

sumption of nickel in plating is making noteable strides especially in Europe. This is being stimulated further by the advent of bright plating. While still in the early stages of application in the United Kingdom and in Europe generally, the process is making substantial progress in the United States. Thus, nickel flows in various forms through an increasing diversity of channels into the industrial processes of the world.

R. H. O.

Sewage Gas.—(*Power Plant Engineering*, Vol. XLI, No. 1.) One of the outstanding applications of sewage gas for power purposes is the installation of Peoria, Ill., which went into service in 1936. The sewage from the Sanitary District of Peoria, after being separated from the liquids, is pumped into large digestion tanks in which the gas is generated. A typical analysis of the gas is: Methane, 60 to 65 per cent.; carbon dioxide, 32 to 38 per cent.; nitrogen, 2 to 3 per cent.; and hydrogen sulphide, 50 to 100 gr. per 100 cu. ft. The net heat content of the gas runs from 500 to 600 B.t.u. per cu. ft. This gas is used as fuel in a 300 hp. gas engine which furnishes power for the pumps and blowers necessary for the operation of the sewage disposal works.

R. H. O.

Elevators with Quota Control.—H. W. WILLIAMS, writing in the *Electric Journal*, Vol. 33, No. 12, states that an improvement in the office building elevator itself will do little to improve service, but a study of the traffic conditions existing in a modern elevator system indicates that it is possible to improve the operation of an elevator bank as a whole. Factors such as loading time, number of stops, and dispatching present a fertile field for materially reducing the round trip time and consequently reducing the number of elevators required, or improving the service with the same number of elevators. The quota control system is one that is fully automatic in which calls are registered by passengers in the corridor and the operator in the car, and the stops are made automatically. Individual stopping relays are provided for each car, and the control is so arranged that the push buttons in the corridors are connected not to a common call registering relay, but to a stopping relay of the individual car that is closest to the floor at the time the call is registered; and the lantern over the door instantly illuminates. In other words, the call is registered definitely on one car only. The number of calls that can be registered on any given car is fixed by a quota to a value which, on an average, will just load the car. When

a car has received its quota, additional calls will be registered on a following car. In this way, the service is divided between all the floors, and an equal service is given to all floors automatically without the necessity of "spotting" cars. The condition is eliminated whereby the upper floors get exceptionally good service and the lower floors poor service. Because of varying traffic conditions, it may be necessary to adjust the quota. This is done by means of a switch at the starter station on the ground floor. Quota control decreases loading time by instantly illuminating the lantern at the particular car entrance to the floor so that passengers will be attracted and be ready to step in the car when it stops.

R. H. O.

Perilla.—This is an oil that has been unappreciated for generations and due to modern chemistry has suddenly become of much interest. According to the *Industrial Bulletin* of Arthur D. Little, Inc. for December 1936 the United States will import from Japan at least 150,000,000 pounds of it this year as against half that much the year before, and a negligible amount before that. The perilla plant is now started as an American crop. The oil has been found of great value in making soya-bean oil useful for paints, varnishes and linoleum in enabling it to dry sufficiently fast. Perilla oil is produced from tiny seeds of a plant of the mint family closely related to the highly colored coleus of our gardens. The plant is an annual which needs a long growing season and does best in the southern states. The Manchurian crop, most of which goes to the United States, is being rapidly increased, and this together with our own raising is expected to make the oil well known fairly soon. The Japanese are as much interested in the press cake as in the oil, and use it as a fertilizer for their mulberry trees. The cake may soon find other than fertilizer uses in this country. Perilla oil alone is not of much interest as a paint oil, for it yellows rather badly in the light (whereas linseed oil yellows in the dark). Further, it has to be boiled or kettle-treated if it is to be used as a paint oil. It is in blends, particularly with soya-bean oil, that perilla achieves the balance of properties that makes it the near sensation it has become. Paints and waterproofings of good drying properties and great durability are possible with this combination of oils from Manchukuo.

R. H. O.